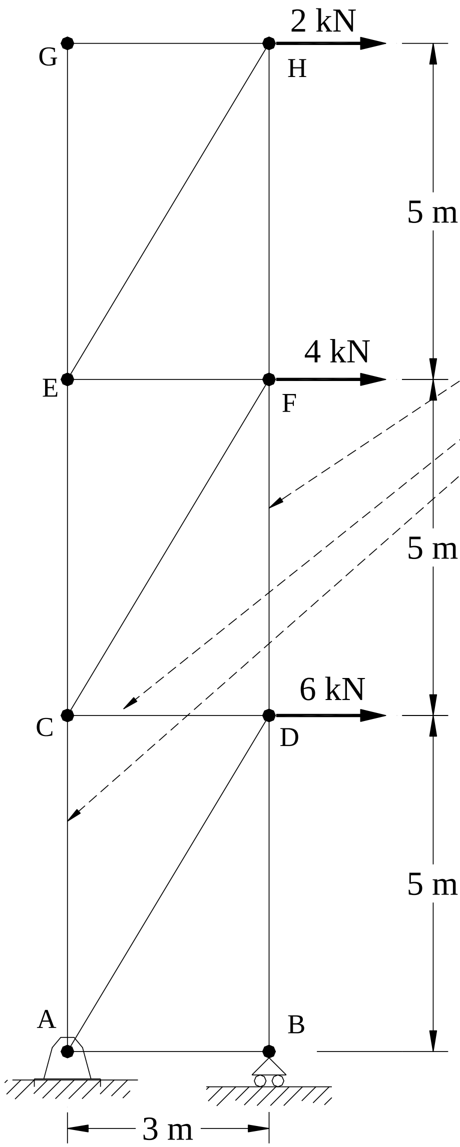
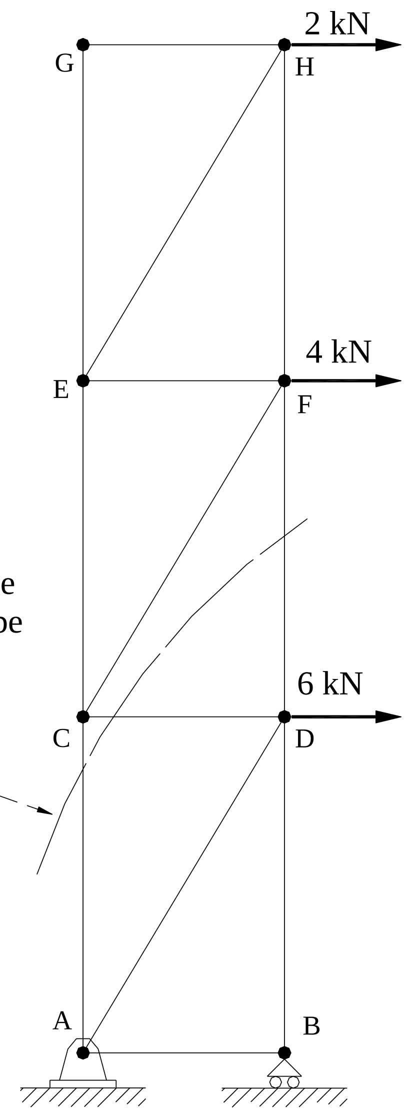
6.3 Trusses: Method of Sections

6.3 Trusses: Method of Sections Example 1, page 1 of 2



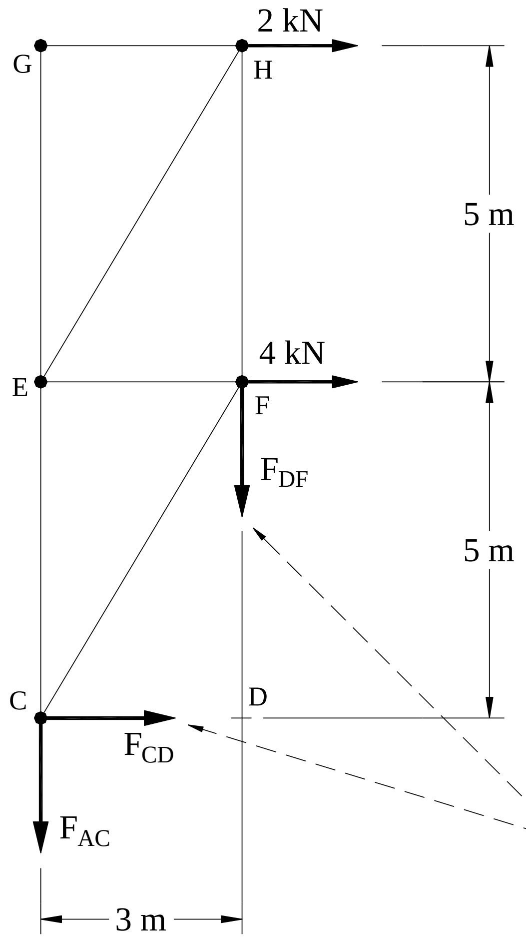
1. Determine the force in members AC, CD, and DF, and state whether the force is tension or compression.



(1) Pass a section through the three members whose forces are to be determined.

6.3 Trusses: Method of Sections Example 1, page 2 of 2

(2) Free-body diagram of portion of truss above the section



(3) At each cut through a member, a force is shown to represent the effect of the portion of the member on one side of the section pulling on the portion on the other side. It is convenient to always assume the force to be tension.

(4) Equations of equilibrium for the portion of the truss (Note that moments are summed about point D, even though point D is *not* part of the free body):

$$\overset{+}{\rightarrow}\Sigma F_x = 0: \;\; 2\text{ kN} + 4\text{ kN} + F_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0: \;\; -F_{AC} - F_{DF} = 0$$

$$\circlearrowleft + \Sigma M_D = 0: \;\; F_{AC}(3\text{ m}) - (2\text{ kN})(5\text{ m} + 5\text{ m}) - (4\text{ kN})(5\text{ m}) = 0$$

Solving simultaneously gives

$F_{AC} = 13.33$ kN (T) ←Ans.

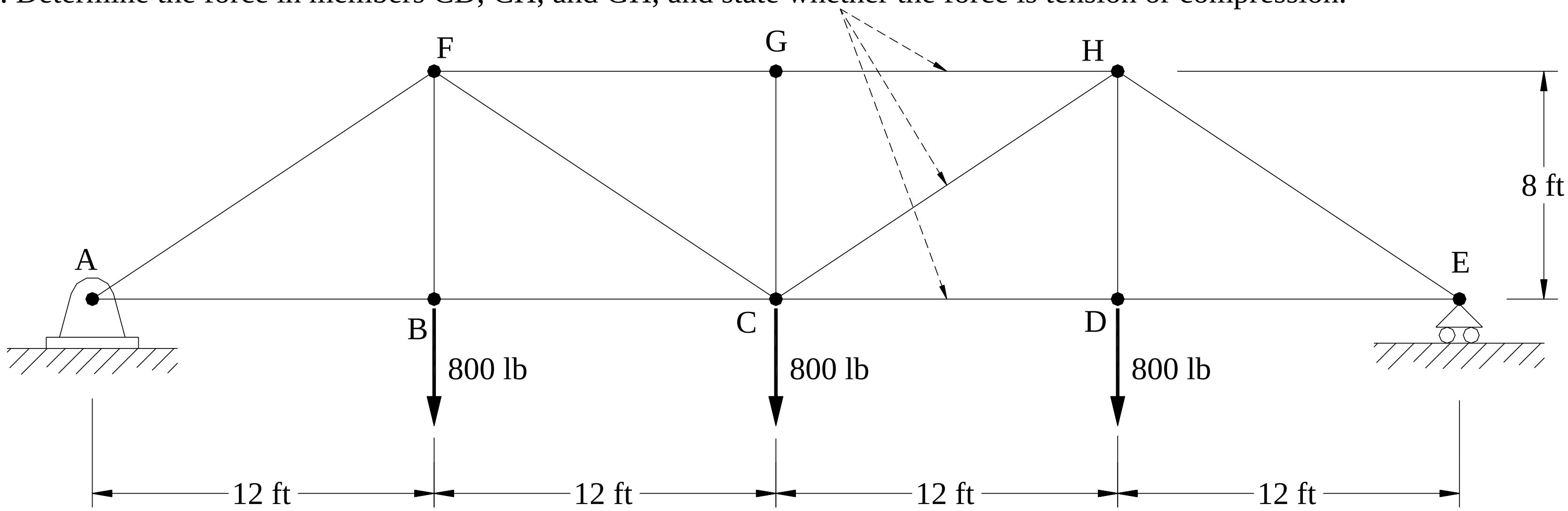
$F_{CD} = -6.0$ kN $= 6.0$ kN (C) ←Ans.

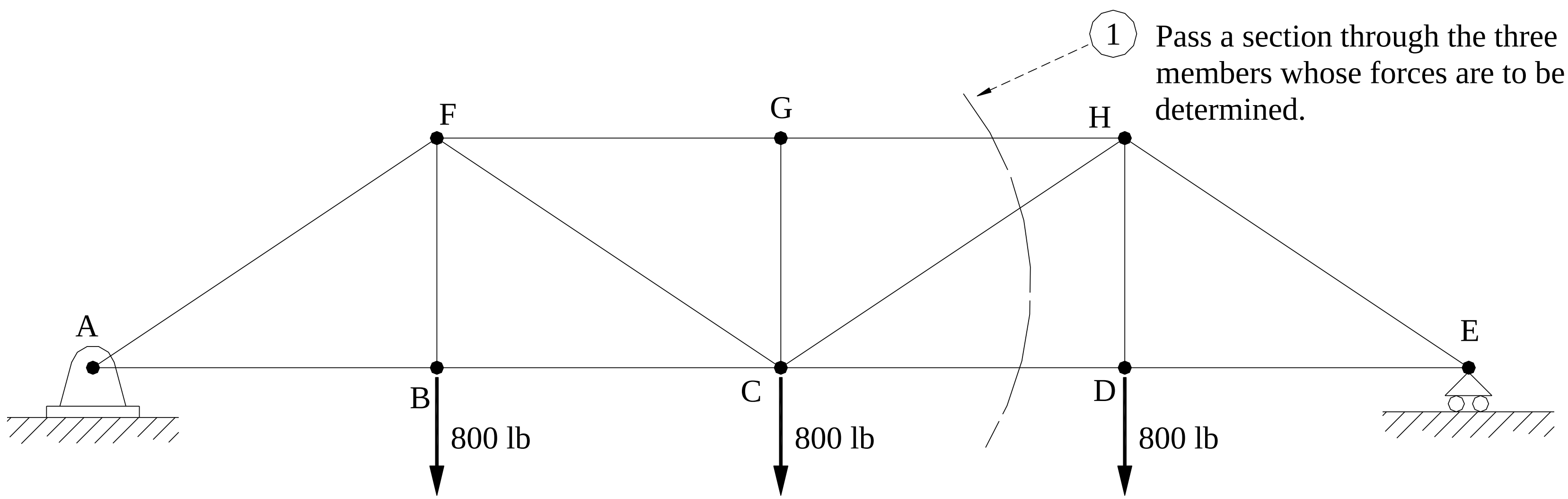
$F_{DF} = -13.33$ kN $= 13.33$ kN (C) ←Ans.

(5) We had assumed member CD to be in tension. Calculations showed that F_{CD} is negative, so our assumption was wrong: CD must be in compression. Similarly DF must be in compression.

6.3 Trusses: Method of Sections Example 2, page 1 of 3

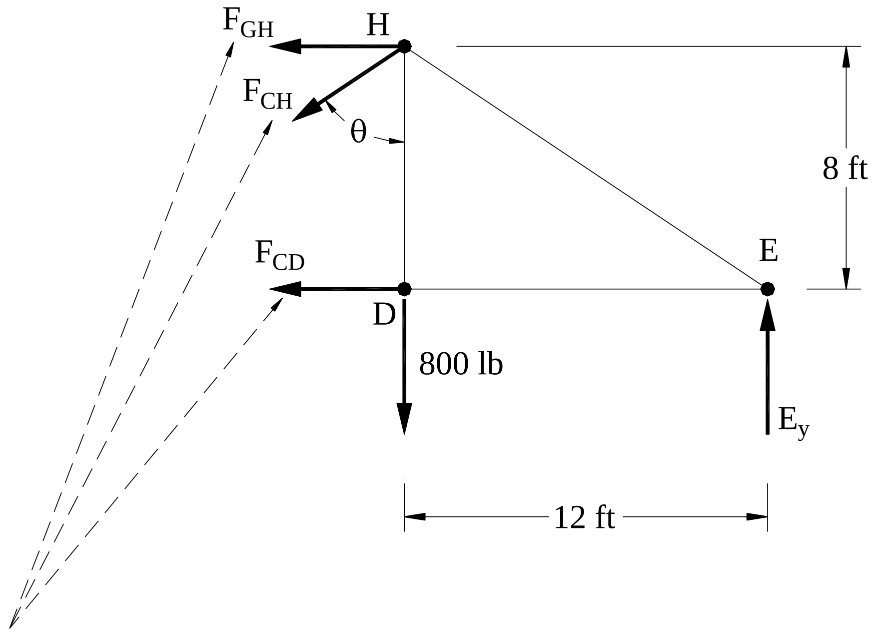
2. Determine the force in members CD, CH, and GH, and state whether the force is tension or compression.





6.3 Trusses: Method of Sections Example 2, page 2 of 3

(2) Free-body diagram of portion of truss to right of section



(3) At each cut through a member, a force is shown.

(4) Equations of equilibrium for the portion of the truss:

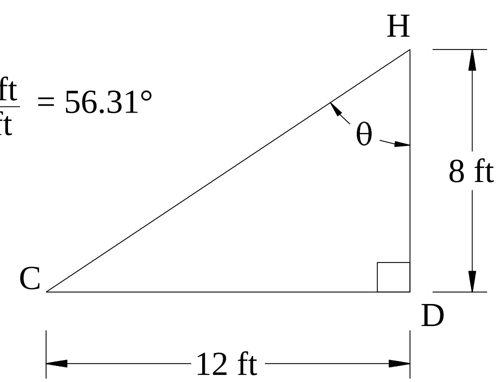
$$\overset{+}{\rightarrow}\Sigma F_x = 0:\ -F_{GH} - F_{CH}\sin\theta - F_{CD} = 0 \qquad (1)$$

$$+\uparrow\Sigma F_y = 0:\ -F_{CH}\cos\theta - 800\text{ lb} + E_y = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_H = 0:\ -F_{CD}\,(8\text{ ft}) + E_y\,(12\text{ ft}) = 0 \qquad (3)$$

(5) Geometry

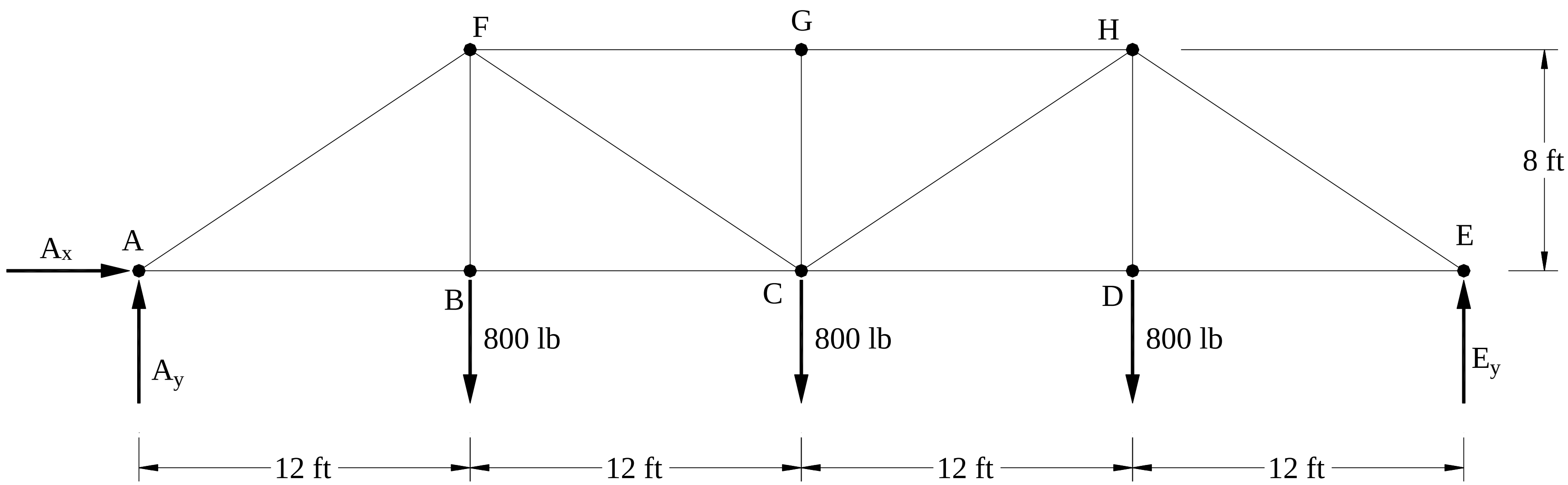
$$\theta = \tan^{-1}\frac{12\text{ ft}}{8\text{ ft}} = 56.31°$$



(6) Three equations but four unknowns, so another equation is needed.

6.3 Trusses: Method of Sections Example 2, page 3 of 3

(7) Free-body diagram of entire truss.



(8) Equilibrium equation for entire truss. This will give the needed fourth equation.

$$\circlearrowleft + \Sigma M_A = 0: \; -(800 \text{ lb})(12 \text{ ft}) - (800 \text{ lb})(2 \times 12 \text{ ft}) - (800 \text{ lb})(3 \times 12 \text{ ft}) + E_y(4 \times 12 \text{ ft}) = 0$$

Solving gives $E_y = 1{,}200$ lb.

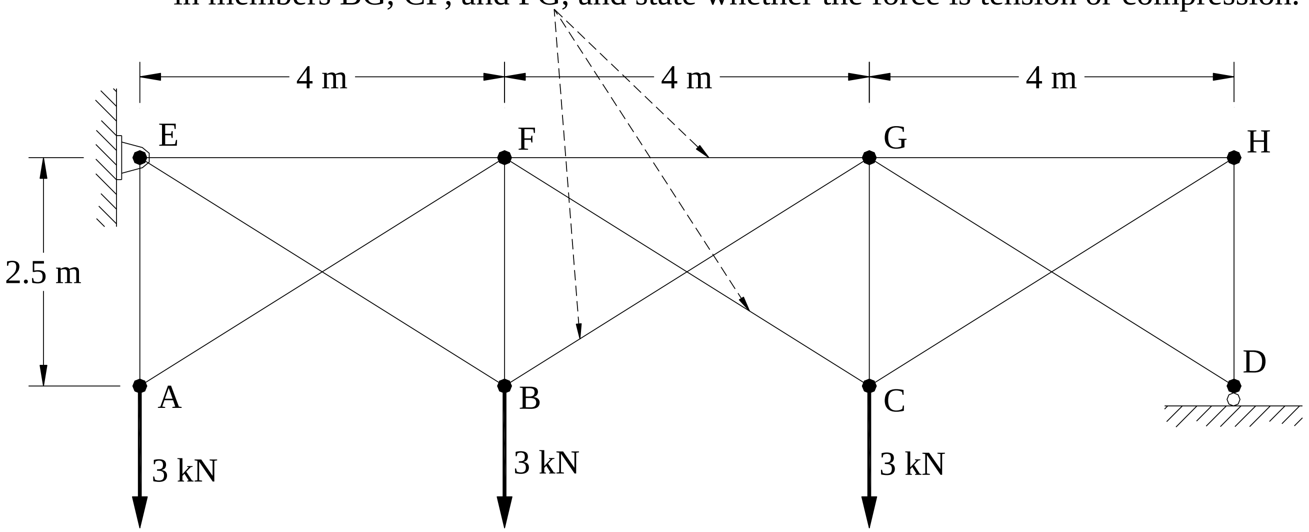
(9) Substituting $E_y = 1{,}200$ lb into Eqs. 1, 2, and 3 and solving simultaneously gives

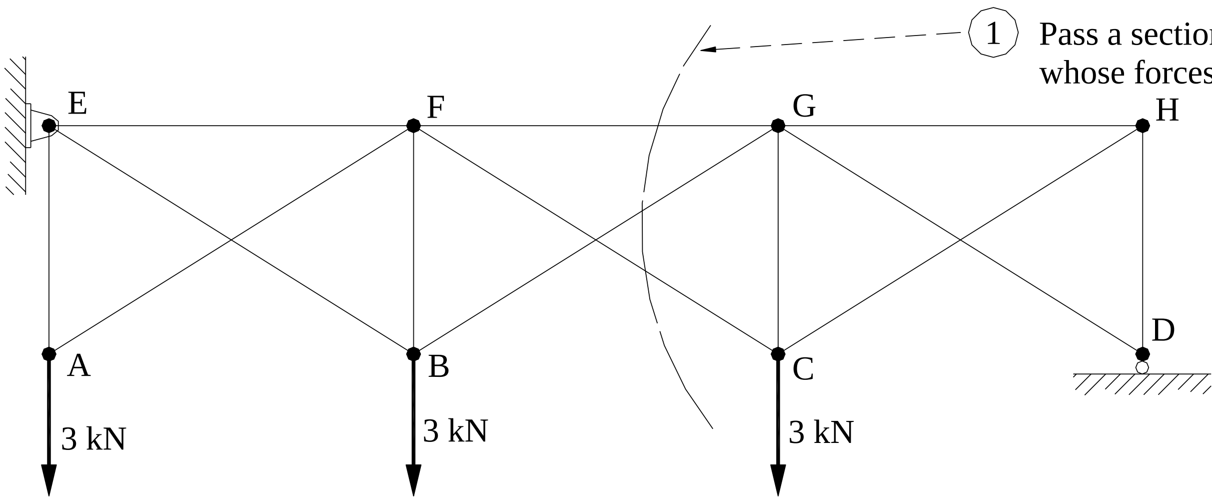
$F_{CD} = 1{,}800$ lb (T) ←Ans.

$F_{CH} = 721$ lb (T) ←Ans.

$F_{GH} = -2{,}400$ lb $= 2{,}400$ lb (C) ←Ans.

3. The diagonal members are not connected to each other where they cross. Determine the force in members BG, CF, and FG, and state whether the force is tension or compression.

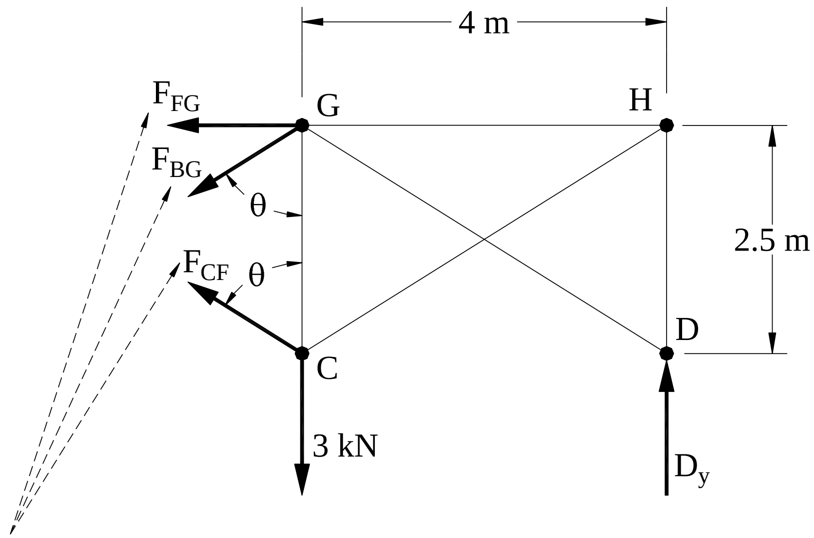




1 Pass a section through the three members whose forces are to be determined.

6.3 Trusses: Method of Sections Example 3, page 2 of 3

(2) Free-body diagram of portion of truss to right of section



(3) At each cut through a member, a force is shown

(4) Equations of equilibrium for the portion of the truss:

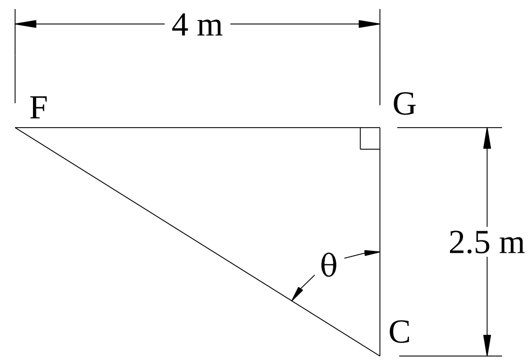
$\overset{+}{\rightarrow}\Sigma F_x = 0$: $-F_{FG} - F_{BG} \sin\theta - F_{CF} \sin\theta = 0$ (1)

$+\uparrow \Sigma F_y = 0$: $-F_{BG} \cos\theta + F_{CF} \cos\theta + D_y - 3\text{ kN} = 0$ (2)

$\circlearrowleft + \Sigma M_G = 0$: $-F_{CF} \sin\theta\,(2.5\text{ m}) + D_y(4\text{ m}) = 0$ (3)

(5) Geometry

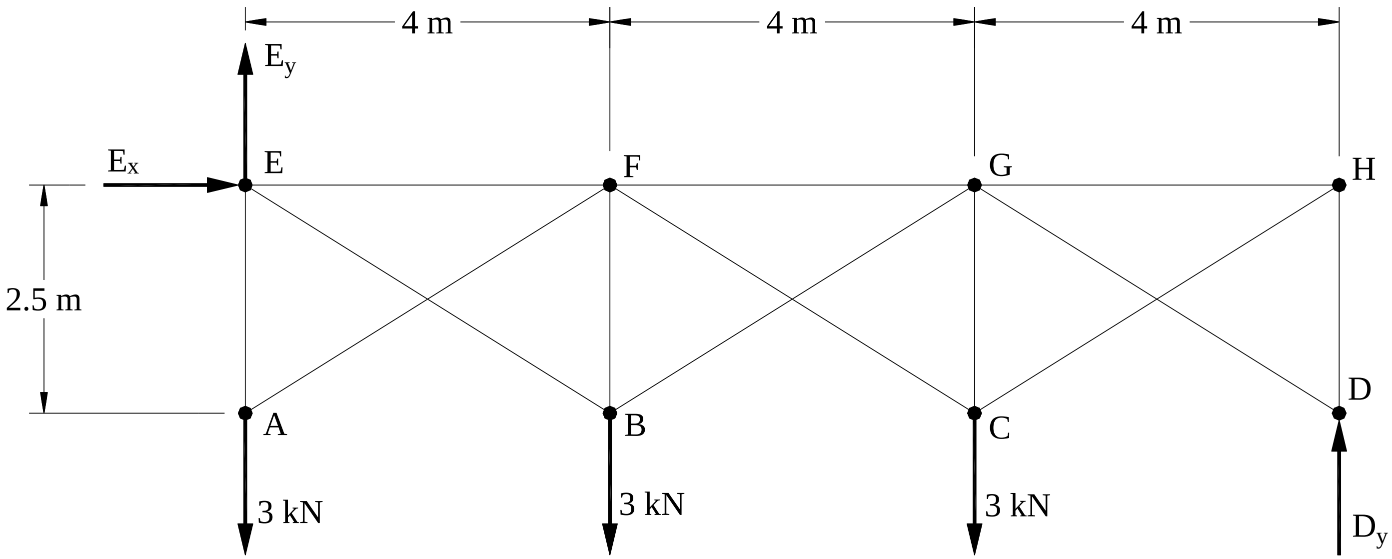
$$\theta = \tan^{-1}\frac{4\text{ m}}{2.5\text{ m}} = 58.0°$$



(6) Three equations but four unknowns, so another equation is needed.

6.3 Trusses: Method of Sections Example 3, page 3 of 3

(7) Free-body diagram of entire truss (This will give the needed fourth equation).



(8) Equilibrium equation for entire truss.

$\circlearrowleft + \Sigma M_E = 0$: $-(3\text{ kN})(4\text{ m}) - (3\text{ kN})(2 \times 4\text{ m}) + D_y(3 \times 4\text{ m}) = 0$

Solving gives $D_y = 3.0$ kN. Then substituting $\theta = 58.0°$ and $D_y = 3.0$ kN into Eqs. 1, 2, and 3 and solving simultaneously gives

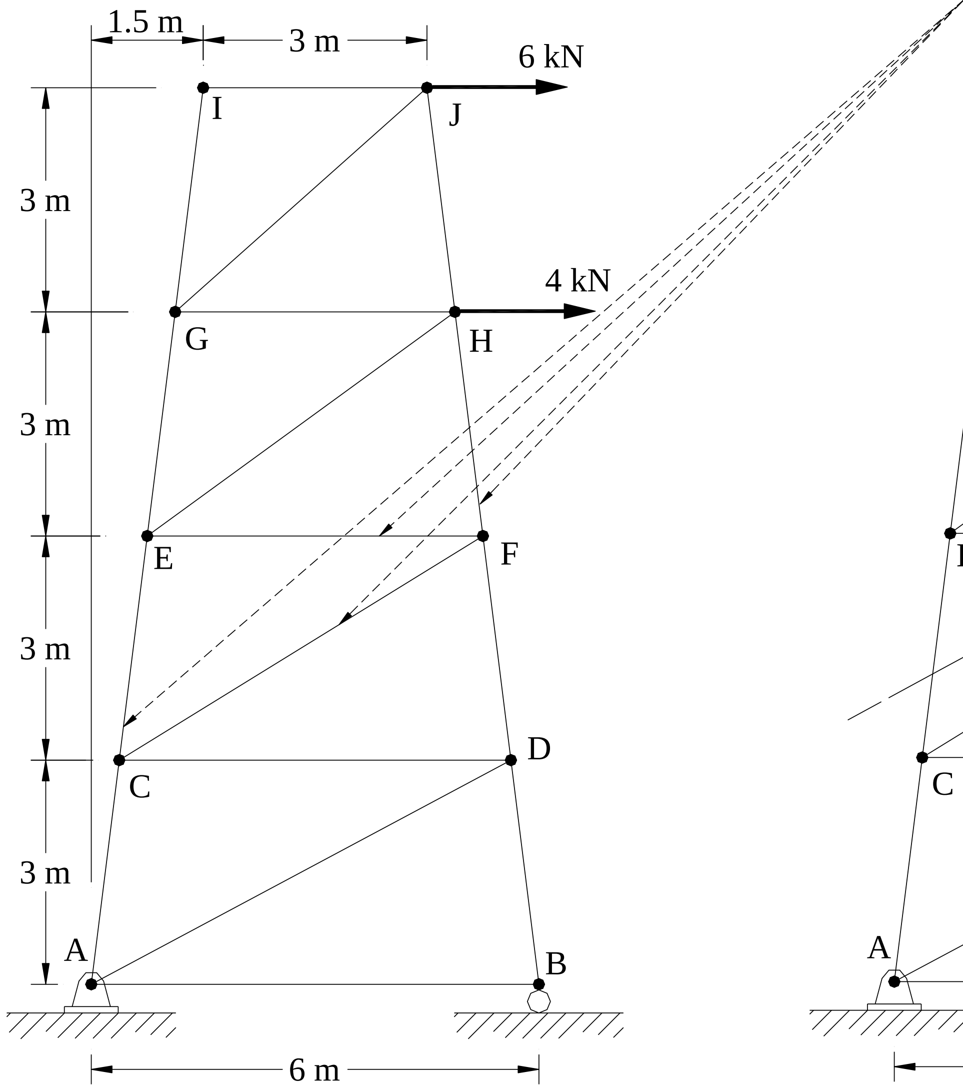
$F_{BG} = 5.66$ kN (T) ←Ans.

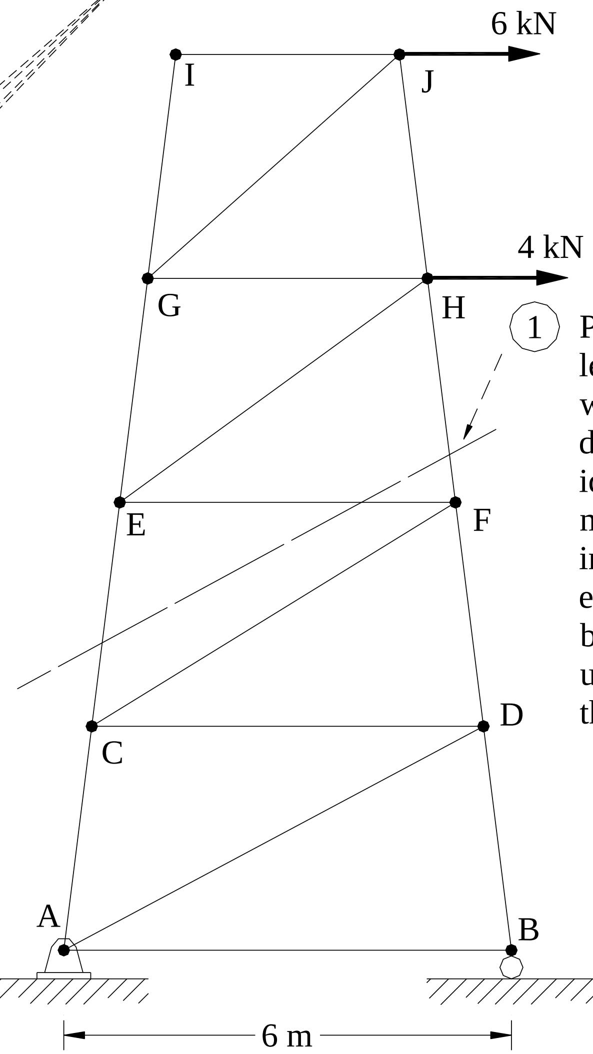
$F_{CF} = 5.66$ kN (T) ←Ans.

$F_{FG} = -9.6$ kN $= 9.6$ kN (C) ←Ans.

6.3 Trusses: Method of Sections Example 4, page 1 of 4

4. Determine the force in members CE, EF, HF, and CF, and state whether the force is tension or compression.

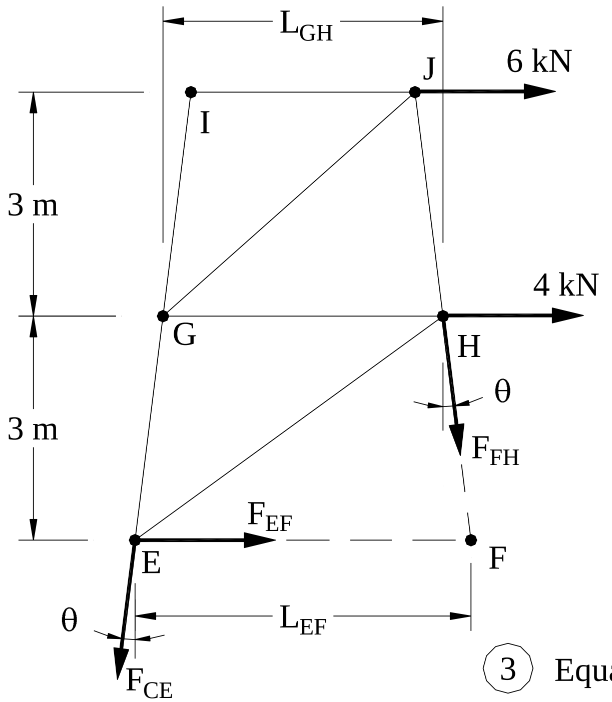




Pass a section through at least some of the members whose forces are to be determined. The general idea is to choose as few members as possible --three in this instance-- because each time a member is cut by a section, an additional unknown is introduced into the equilibrium equations.

6.3 Trusses: Method of Sections Example 4, page 2 of 4

(2) Free-body diagram of portion of truss above section (Using the upper portion of the truss rather than the lower eliminates the need to calculate the reactions at the bottom of the truss).



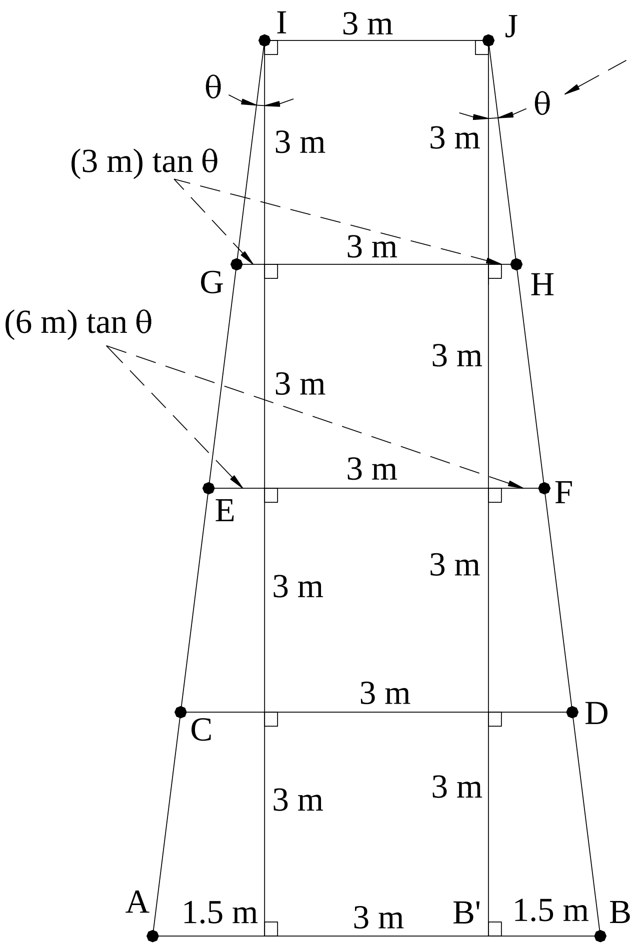
(3) Equations of equilibrium for the portion of the truss:

$$\circlearrowleft + \Sigma M_G = 0: \ -(6 \text{ kN})(3 \text{ m}) + F_{EF}(3 \text{ m}) - F_{FH} \cos\theta \, (L_{GH}) = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_F = 0: \ -(6 \text{ kN})(2 \times 3 \text{ m}) - (4 \text{ kN})(3 \text{ m}) + F_{CE} \cos\theta \, (L_{EF}) = 0 \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = 0: \ -F_{CE} \sin\theta + F_{EF} + F_{FH} \sin\theta + 4 \text{ kN} + 6 \text{ kN} = 0 \qquad (3)$$

(4) Geometry



(5) $\theta = \tan^{-1}\dfrac{BB'}{JB'} = \tan^{-1}\dfrac{1.5\text{ m}}{4 \times 3\text{ m}} = 7.125°$

$L_{GH} = 3\text{ m} + (3\text{ m})\tan\theta + (3\text{ m})\tan\theta = 3.75\text{ m}$

$L_{EF} = 3\text{ m} + (6\text{ m})\tan\theta + (6\text{ m})\tan\theta = 4.50\text{ m}$

(6) Substituting these values for θ, L_{GH}, and L_{EF} into Eqs. 1, 2, and 3 and solving simultaneously gives:

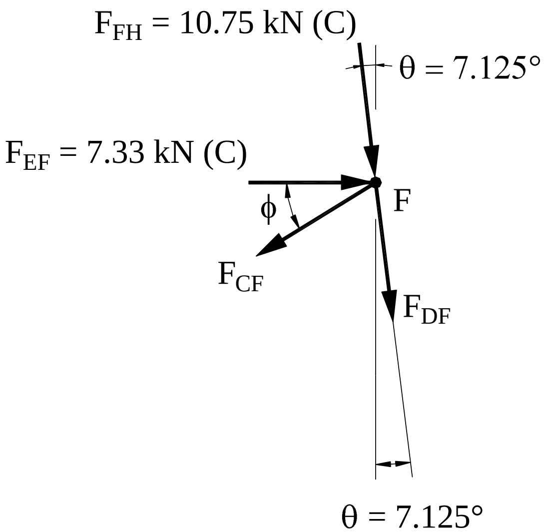
$F_{CE} = 10.75\text{ kN}$ ←Ans.

$F_{EF} = -7.33\text{ kN} = 7.33\text{ kN (C)}$ ←Ans.

$F_{FH} = -10.75\text{ kN} = 10.75\text{ kN (C)}$ ←Ans.

6.3 Trusses: Method of Sections Example 4, page 4 of 4

(7) Free-body diagram of joint F. This free body will enable us to calculate the remaining unknown force—the force in member CF.

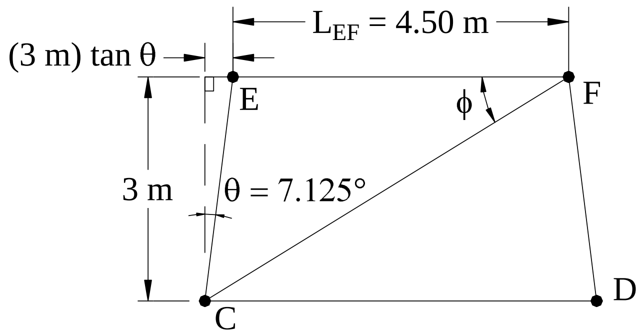


(8) Equilibrium equations for joint F

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \; -F_{CF} \cos \phi + F_{DF} \sin 7.125° + (10.75 \text{ kN})(\sin 7.125°) + 7.33 \text{ kN} = 0 \qquad (4)$$

$$+\uparrow \Sigma F_y = 0: \; -F_{CF} \sin \phi - F_{DF} \cos 7.125° - (10.75 \text{ kN})(\cos 7.125°) = 0 \qquad (5)$$

(9) Geometry



$$\phi = \tan^{-1} \frac{3 \text{ m}}{(3 \text{ m}) \tan 7.125° + 4.50 \text{ m}} = 31.61°$$

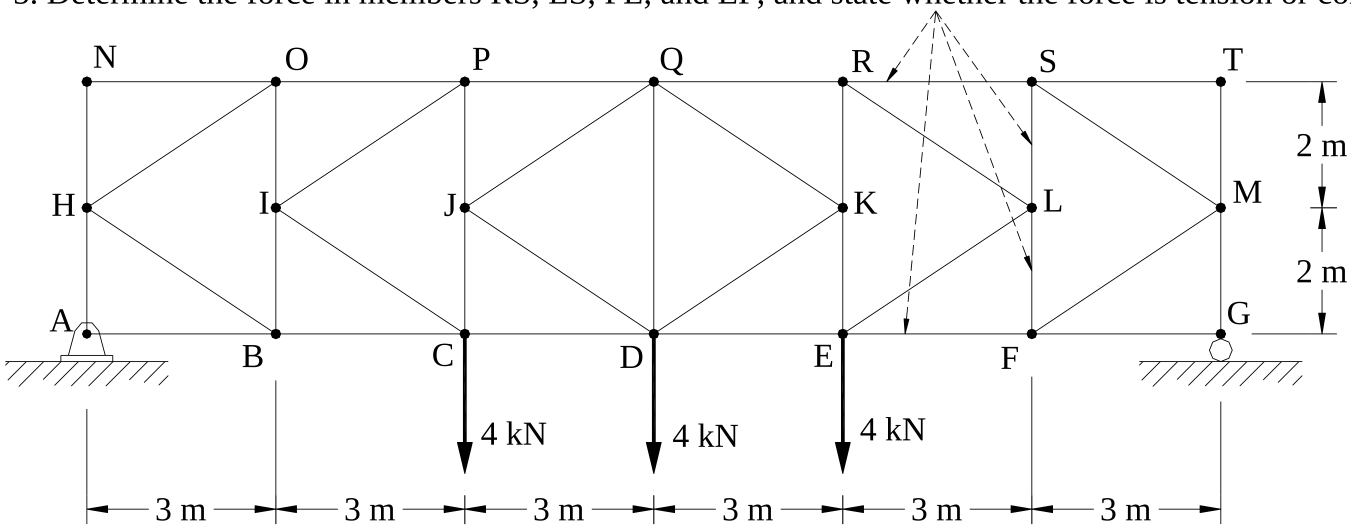
(10) Substituting $\phi = 31.608°$ into Eqs. 4 and 5 and solving simultaneously gives:

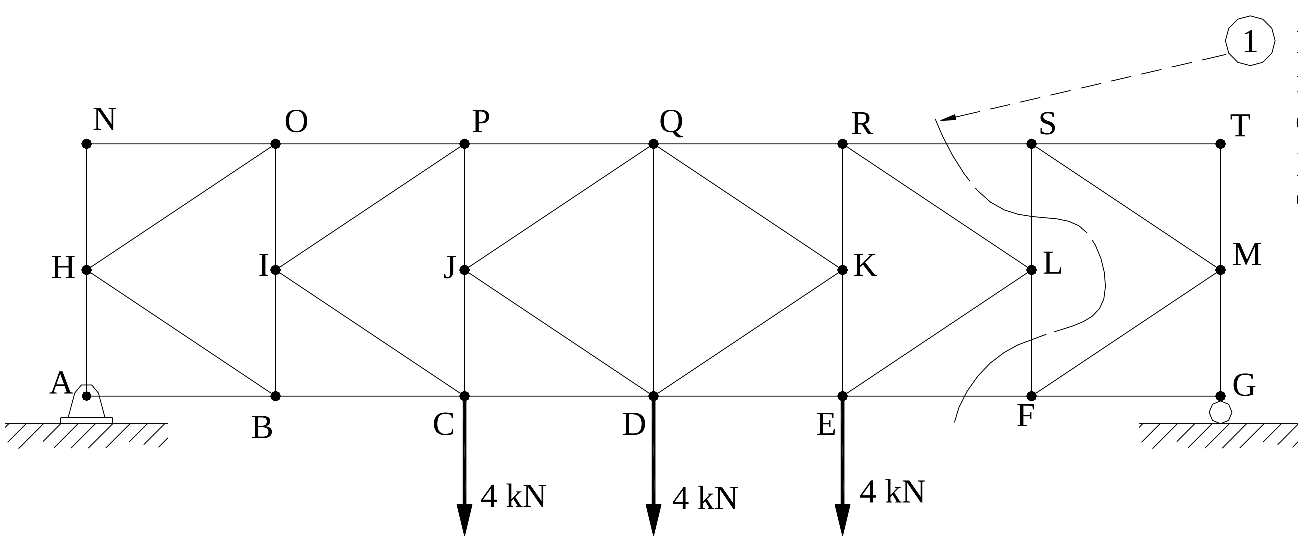
$F_{DF} = -14.97$ kN = 14.97 kN (C)

$F_{CF} = 7.99$ kN (T) ←Ans.

6.3 Trusses: Method of Sections Example 5, page 1 of 4

5. Determine the force in members RS, LS, FL, and EF, and state whether the force is tension or compression.

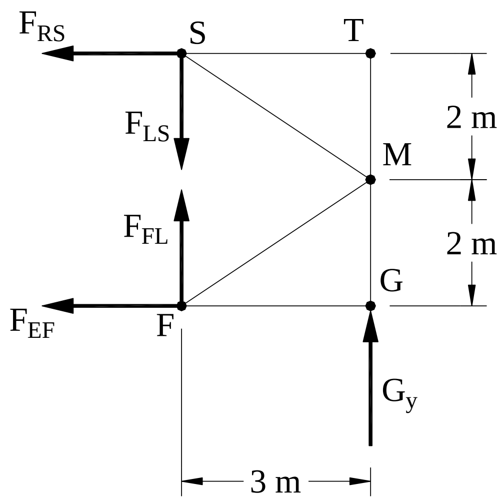




1. Pass a section through the four members whose forces are to be determined. It does not appear possible to find a section that cuts only three of these members.

6.3 Trusses: Method of Sections Example 5, page 2 of 4

2 Free body diagram of truss portion to right of section line



3 Equations of equilibrium for the portion of the truss:

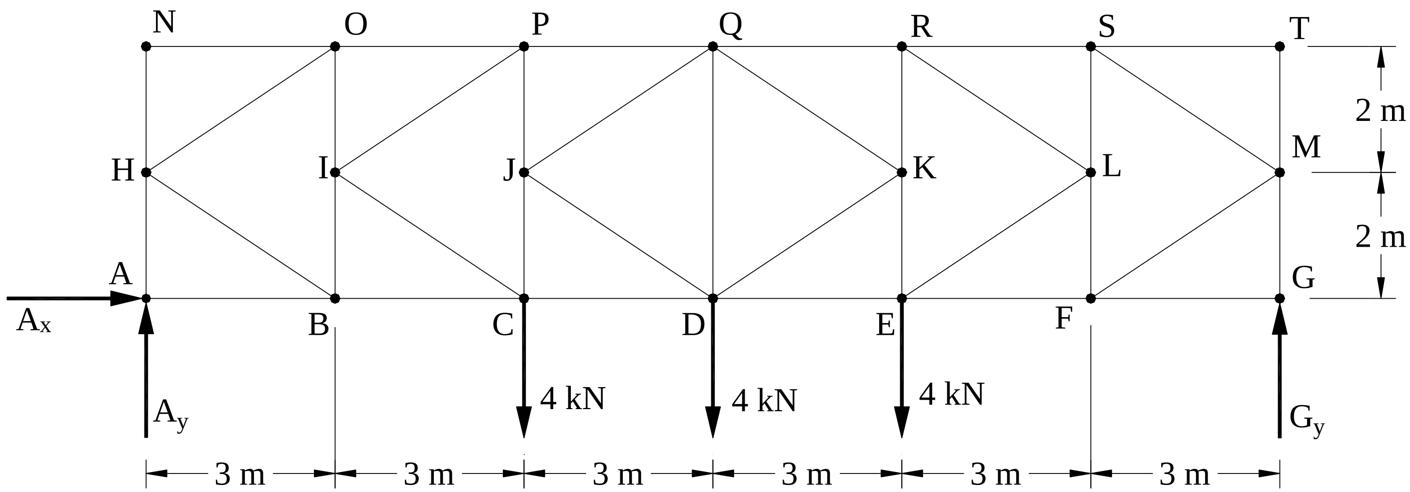
$\circlearrowleft + \Sigma M_S = 0$: $-F_{EF}(2 \times 2\text{ m}) + G_y(3\text{ m}) = 0$ (1)

$\circlearrowleft + \Sigma M_F = 0$: $F_{RS}(2 \times 2\text{ m}) + G_y(3\text{ m}) = 0$ (2)

$+\uparrow \Sigma F_y = 0$: $F_{FL} - F_{LS} + G_y = 0$ (3)

4 Three equations with five unknowns so two more equations are needed.

5 Free-body diagram of entire truss (This free body will enable us to calculate the reaction at G).



6 Equation of equilibrium for the entire truss.

$$\circlearrowleft + \Sigma M_A = 0: \; -(4 \text{ kN})(2 \times 3 \text{ m}) - (4 \text{ kN})(3 \times 3 \text{ m}) - (4 \text{ kN})(4 \times 3 \text{ m}) + G_y(18 \text{ m}) = 0 \qquad (4)$$

Solving gives

$G_y = 6$ kN

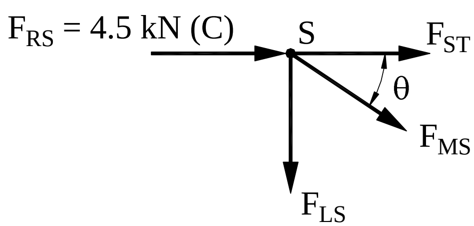
Substituting $G_y = 6$ kN into Eqs. 1 and 2 and solving gives:

$F_{EF} = 4.5$ kN (T) ←Ans.

$F_{RS} = -4.5$ kN $= 4.5$ kN (C) ←Ans.

6.3 Trusses: Method of Sections Example 5, page 4 of 4

(7) Free-body diagram of joint S. This free body will enable us to calculate the force in member LS.

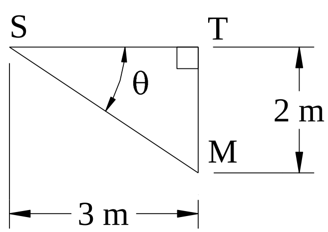


(8) Equations of equilibrium for joint S. Note that there are three unknowns but only two equations.

$$\xrightarrow{+} \Sigma F_x = 0: \; 4.5\text{ kN} + F_{ST} + F_{MS}\cos\theta = 0 \qquad (5)$$

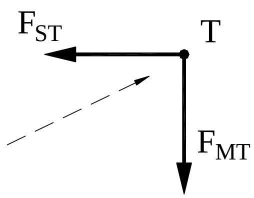
$$+\uparrow \Sigma F_y = 0: \; -F_{LS} - F_{MS}\sin\theta = 0 \qquad (6)$$

(9) Geometry



$$\theta = \tan^{-1}\frac{2\text{ m}}{3\text{ m}} = 33.69°$$

(10) Free body diagram of joint T



(11) Two members meet at joint T, they are not collinear and no external force acts at joint T, so members ST and MT are zero-force members.

Substituting $F_{ST} = 0$ in Eq. 5 and solving Eqs. 5 and 6 simultaneously gives:

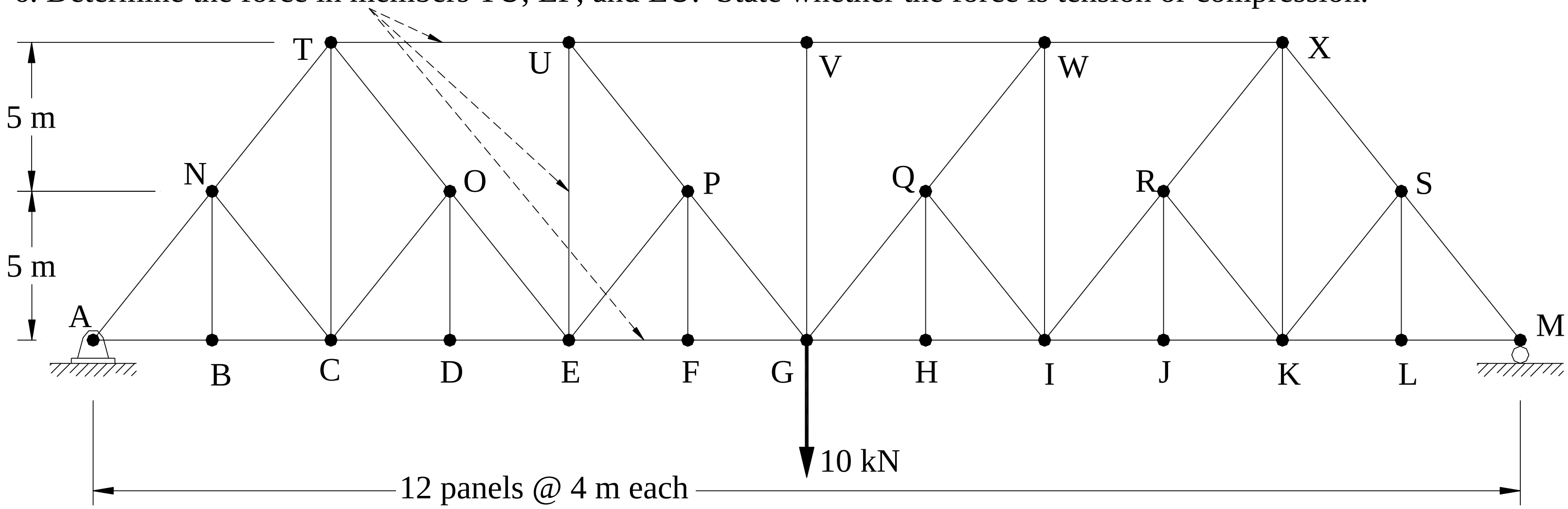
$$F_{MS} = -5.41\text{ kN} = 5.41\text{ kN (C)}$$

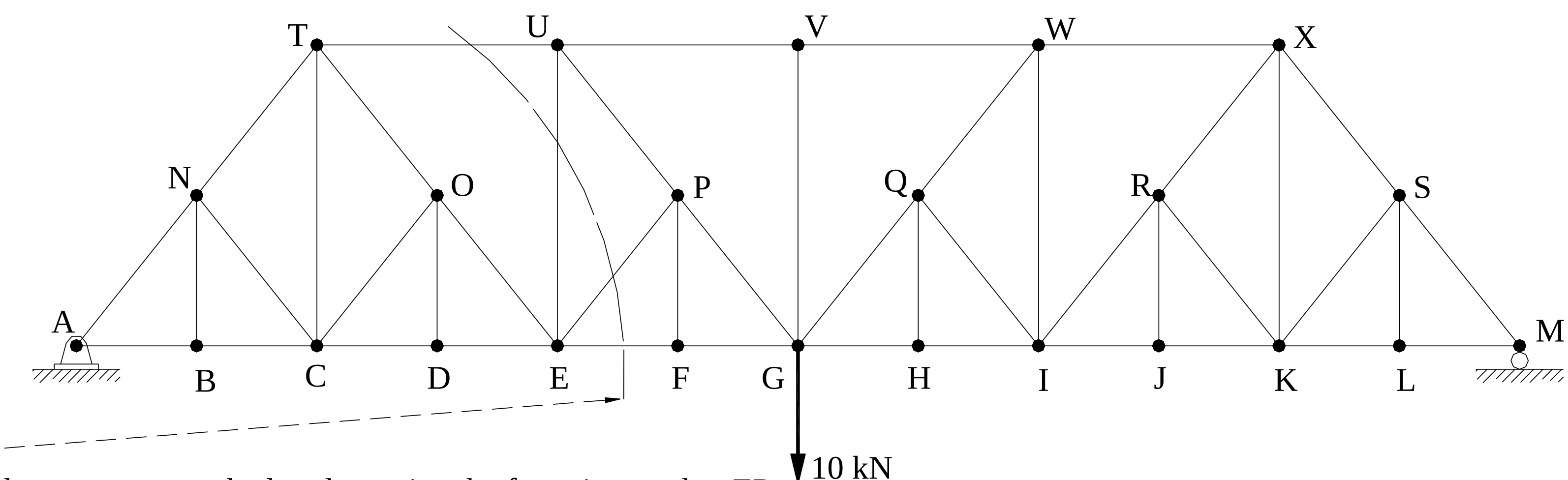
$$F_{LS} = 3.0\text{ kN (T)} \qquad \leftarrow\text{Ans.}$$

(12) Substituting F_{LS} = 3.0 kN and G_y = 6 kN into Eq. 3 and solving gives:

$$F_{FL} = -3.0\text{ kN} = 3.0\text{ kN (C)} \qquad \leftarrow\text{Ans.}$$

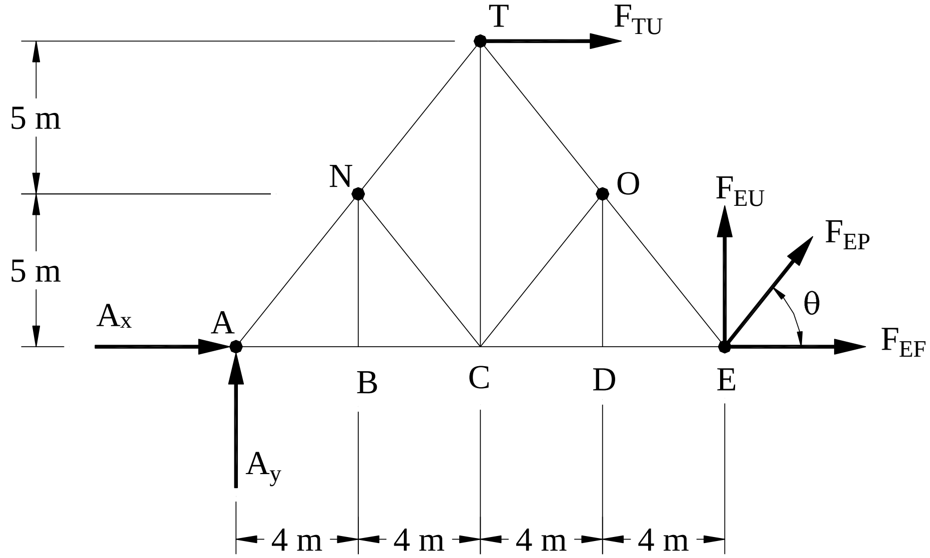
6. Determine the force in members TU, EF, and EU. State whether the force is tension or compression.





1 Even though we were not asked to determine the force in member EP, we have to pass the section through it because we must make the section go completely through the truss.

(2) Free-body diagram of portion of the truss to the left of the section



(3) Equations of equilibrium for the portion of the truss:

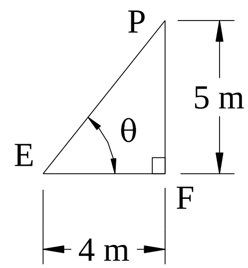
$\overset{+}{\rightarrow} \Sigma F_x = 0$: $A_x + F_{TU} + F_{EP} \cos\theta + F_{EF} = 0$ (1)

$+\uparrow \Sigma F_y = 0$: $A_y + F_{EU} + F_{EP} \sin\theta = 0$ (2)

$\circlearrowleft + \Sigma M_E = 0$: $-A_y (4 \times 4 \text{ m}) - F_{TU}(2 \times 5 \text{ m}) = 0$ (3)

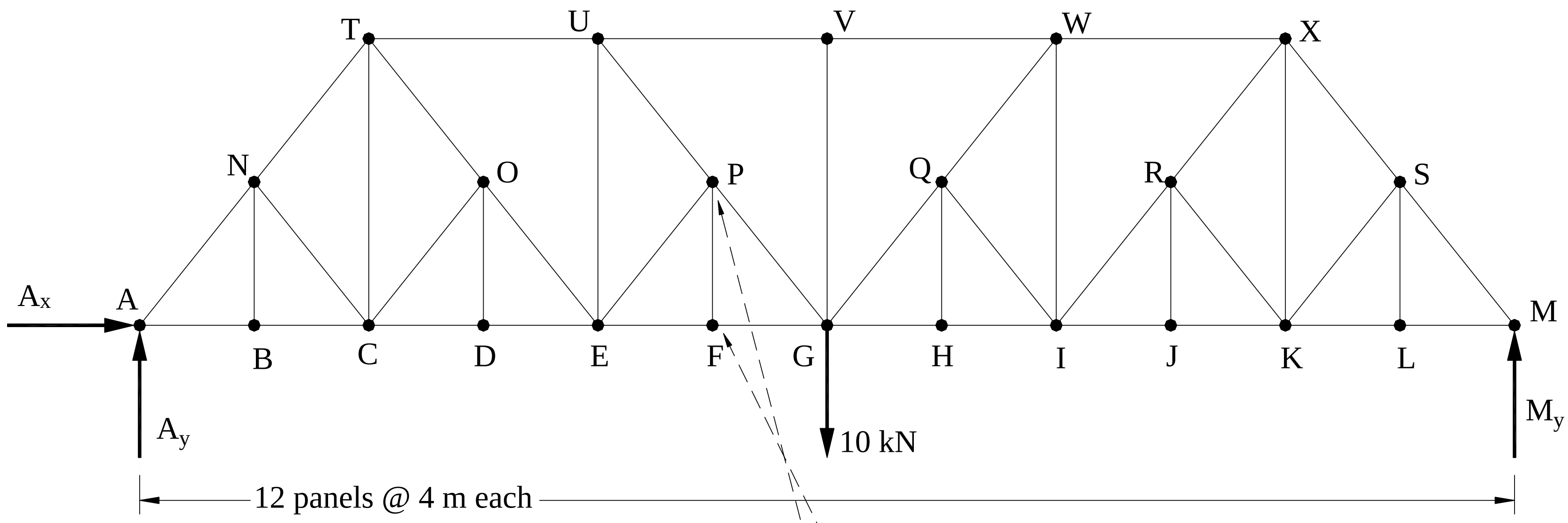
(4) Three equations with six unknowns so three more equations are needed.

(5) Geometry



$$\theta = \tan^{-1}\frac{5 \text{ m}}{4 \text{ m}} = 51.34°$$

(6) Free-body diagram of entire truss (This free body will enable us to calculate the reactions at support A).



(7) Equations of equilibrium for entire truss. Note that we only write *two* equations because we only need to calculate Ax and Ay, since only Ax and Ay appear in Eqs. 1, 2, and 3.

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $A_x = 0$ (4)

$\circlearrowleft + \Sigma M_M = 0$: $(10\text{ kN})(6 \times 4\text{ m}) - A_y(12 \times 4\text{ m}) = 0$ (5)

Solving gives $A_x = 0$ and $A_y = 5$ kN.

(8) Consideration of joint F shows that member FP is a zero-force member, so $F_{FP} = 0$.

But if member FP is removed (because it is a zero-force member), consideration of joint P shows that member EP is also a zero-force member, so $F_{EP} = 0$.

6.3 Trusses: Method of Sections Example 6, page 4 of 4

(9) Substituting

$\theta = 51.34°$,

$A_x = 0$,

$A_y = 5$ kN,

and

$F_{EP} = 0$

into Eqs. 1, 2, and 3, and solving gives:

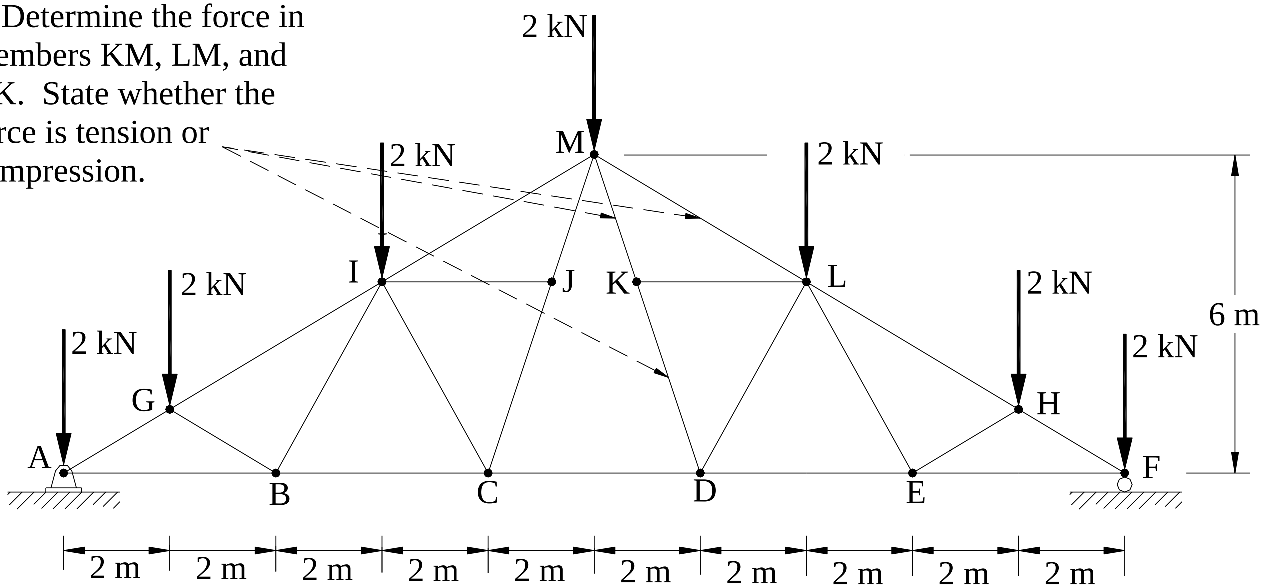
$F_{TU} = –8$ kN = 8 kN (C) ←Ans.

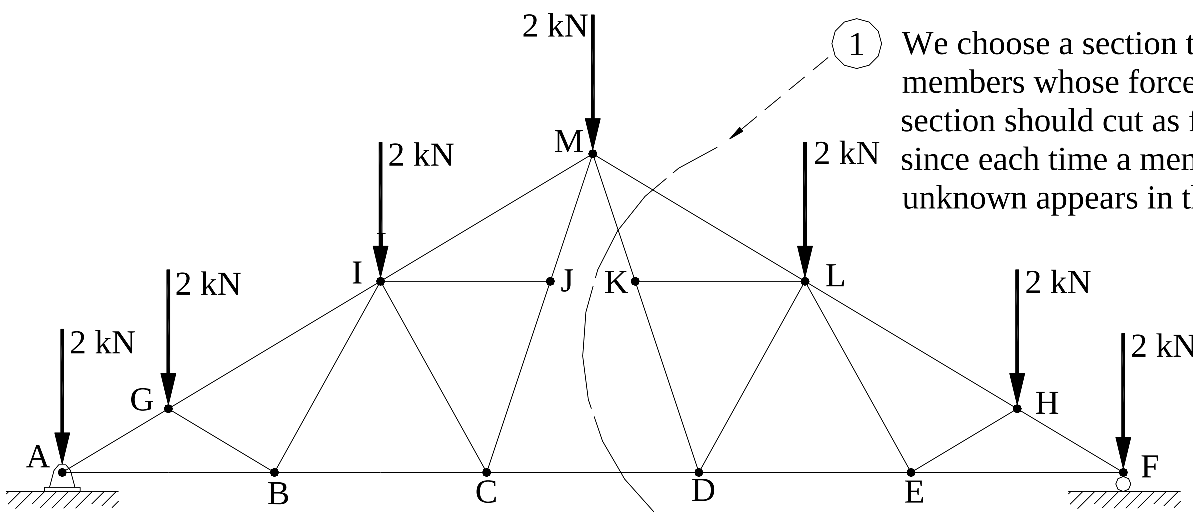
$F_{EF} = 8$ kN (T) ←Ans.

$F_{EU} = –5$ kN = 5 kN (C) ←Ans.

6.3 Trusses: Method of Sections Example 7, page 1 of 6

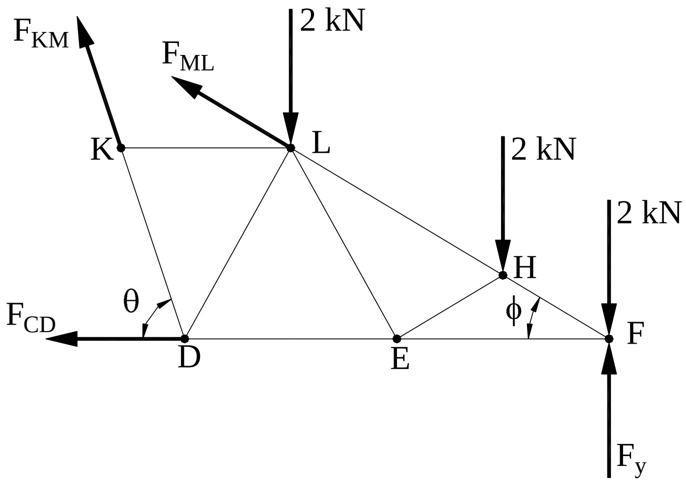
7. Determine the force in members KM, LM, and DK. State whether the force is tension or compression.



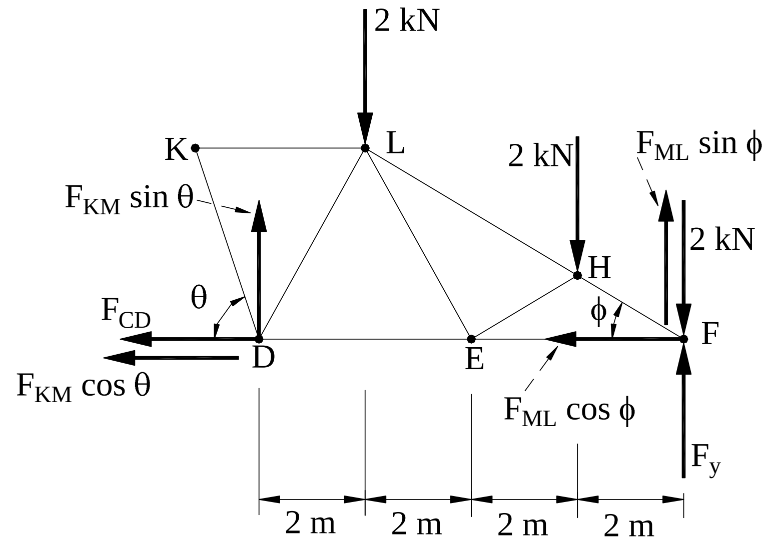


We choose a section that cuts at least some of the members whose forces are to be determined. But the section should cut as few other members as possible, since each time a member is cut, an additional unknown appears in the equilibrium equations.

2. Free-body diagram of portion of truss to right of section. It is not essential but we can save some work if we use the principle of transmissibility as shown in Step 3.



3. Same free body as in Step 2, but now the force F_{KM} has been moved along its line of action to joint D (principle of transmissibility) and then expressed in terms of vertical and horizontal components. Similarly F_{ML} is moved to joint F.



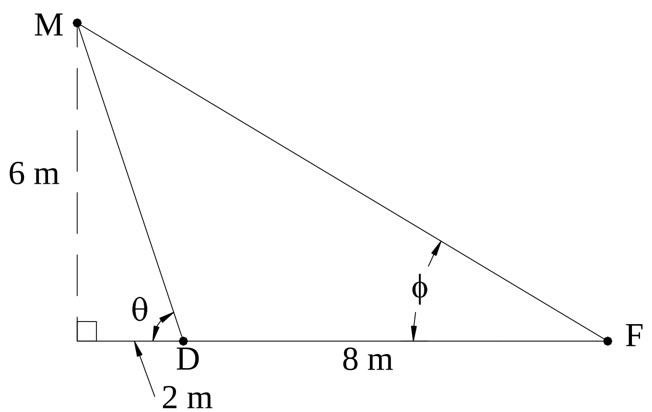
4. Equations of equilibrium for free body in Step 3. Note that because we were not asked to determine F_{CD}, we choose two moment equations in which F_{CD} does not appear.

$$\circlearrowleft + \Sigma M_F = 0: \quad (2\text{ kN})(2\text{ m}) + (2\text{ kN})(3 \times 2\text{ m}) - F_{KM} \sin\theta\,(4 \times 2\text{ m}) = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_D = 0: \quad -(2\text{ kN})(2\text{ m}) - (2\text{ kN})(3 \times 2\text{ m}) - (2\text{kN})(4 \times 2\text{ m}) + F_y(4 \times 2\text{ m}) + F_{ML} \sin\phi\,(4 \times 2\text{ m}) = 0 \qquad (2)$$

5. Two equations but three unknown forces, so another equilibrium equation is needed.

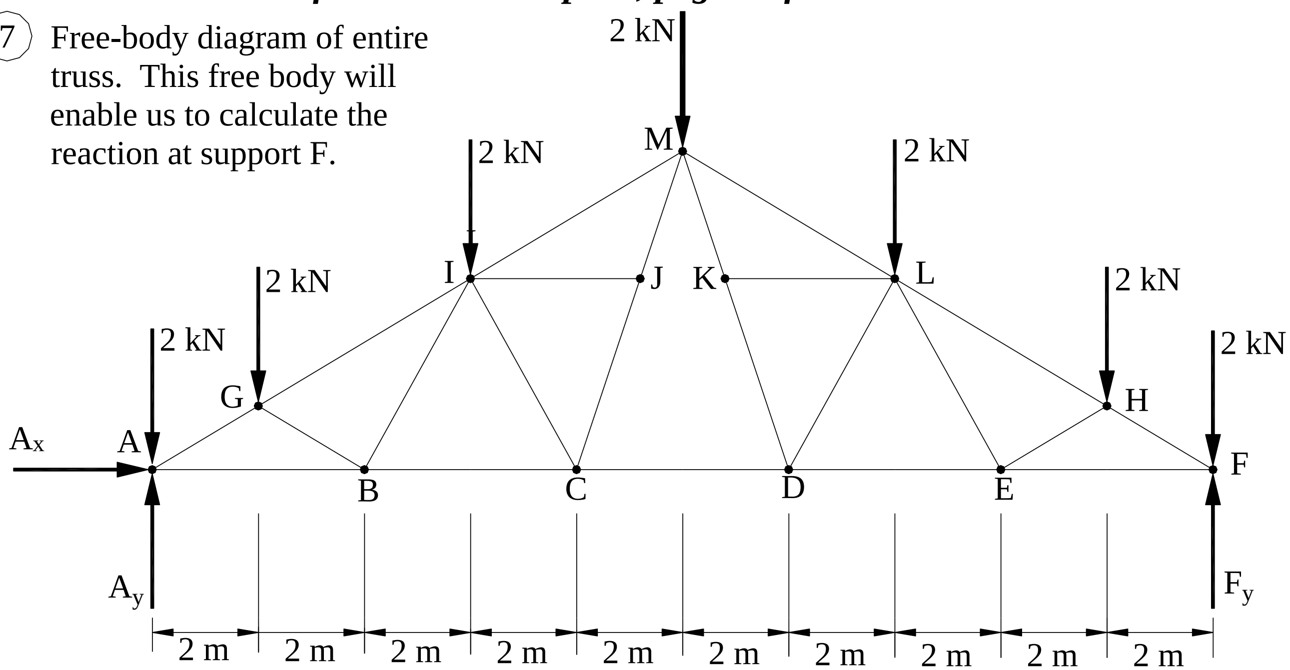
6 Geometry



$$\theta = \tan^{-1}\left(\frac{6\text{ m}}{2\text{ m}}\right) = 71.56°$$

$$\phi = \tan^{-1}\left(\frac{6\text{ m}}{2\text{ m} + 8\text{ m}}\right) = 30.96°$$

(7) Free-body diagram of entire truss. This free body will enable us to calculate the reaction at support F.



(8) Equilibrium equation for entire truss

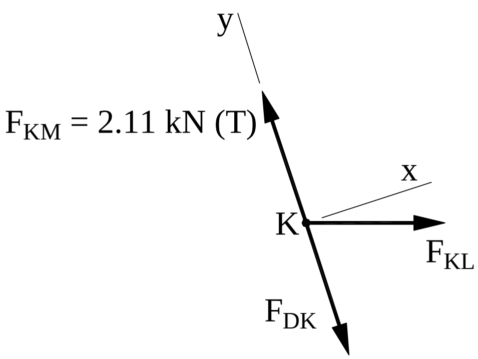
$$\circlearrowleft +\Sigma M_A = 0:\ -(2\text{ kN})(2\text{ m}) - (2\text{ kN})(3 \times 2\text{ m}) - (2\text{ kN})(5 \times 2\text{ m})$$
$$- (2\text{ kN})(7 \times 2\text{ m}) - (2\text{ kN})(9 \times 2\text{ m}) - (2\text{ kN})(10 \times 2\text{ m}) + F_y(10 \times 2\text{ m}) = 0 \qquad (3)$$

(9) Solving gives $F_y = 7$ kN. Substituting $F_y = 7$ kN, $\theta = 71.56°$, and $\phi = 30.96°$ in Eqs. 1 and 2, and solving simultaneously gives:

$F_{KM} = 2.11$ kN (T) ←Ans.

$F_{ML} = -5.83$ kN $= 5.83$ kN (C) ←Ans.

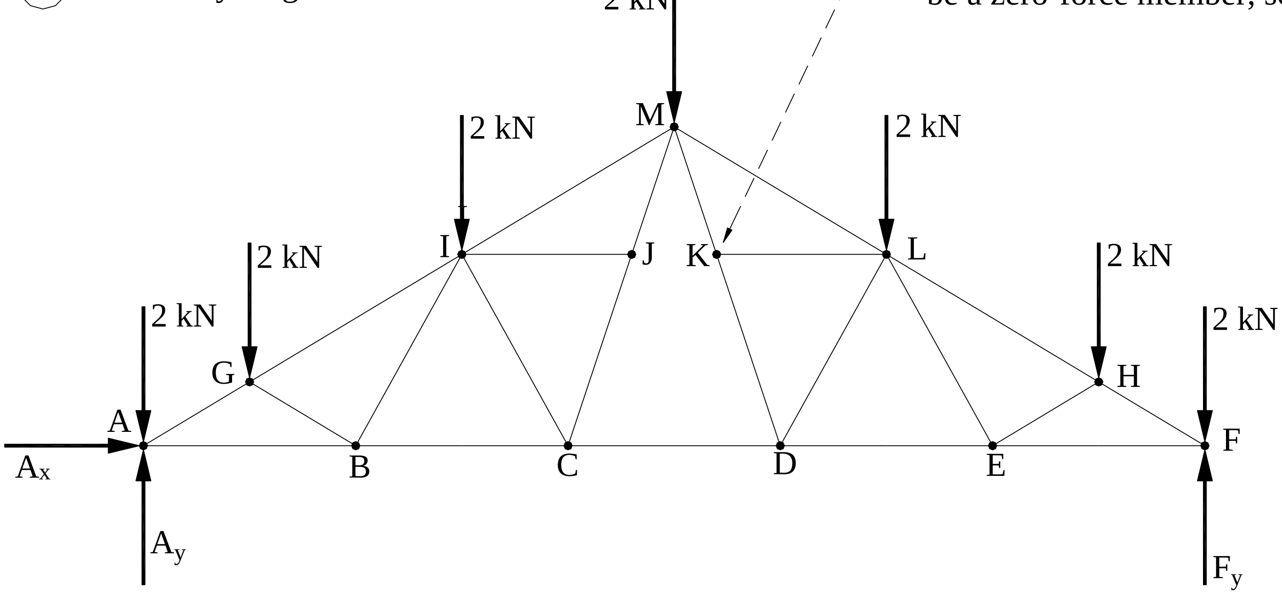
(10) Free-body diagram of joint K (This free body will enable us to calculate the force in member DK).



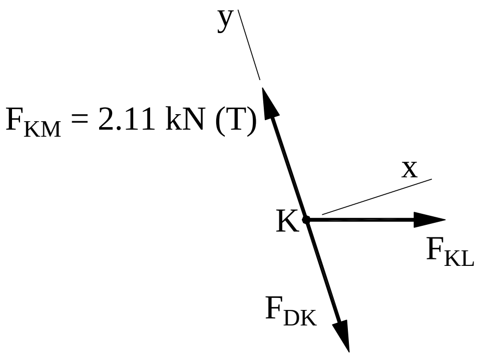
(11) Since there are only two unknown forces, F_{KL} and F_{DK}, we could write force-equilibrium equations in the x and y directions and then solve them simultaneously. However, we can save work by noticing that a zero-force member is present.

(12) Free- body diagram of entire truss

(13) Consideration of joint K shows that KL must be a zero-force member, so F_{KL} = 0.



(14) Free-body diagram of joint K (repeated)

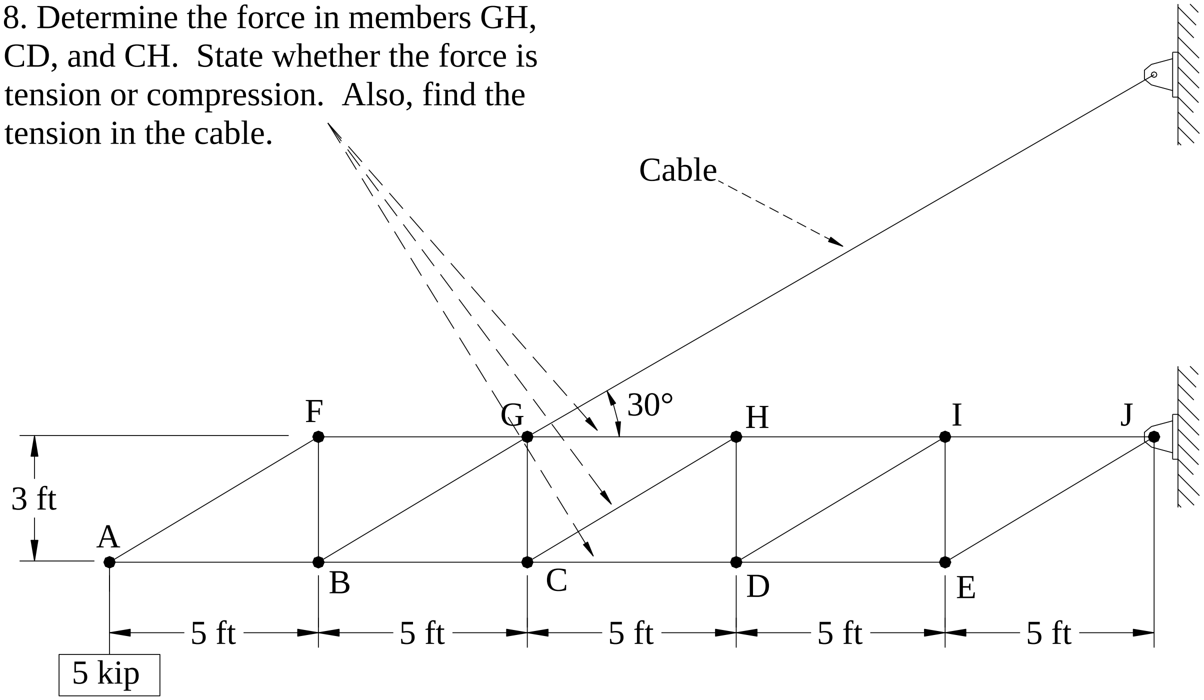


$+\nwarrow \Sigma F_y = 2.11 \text{ kN} - F_{DK} = 0$ (4)

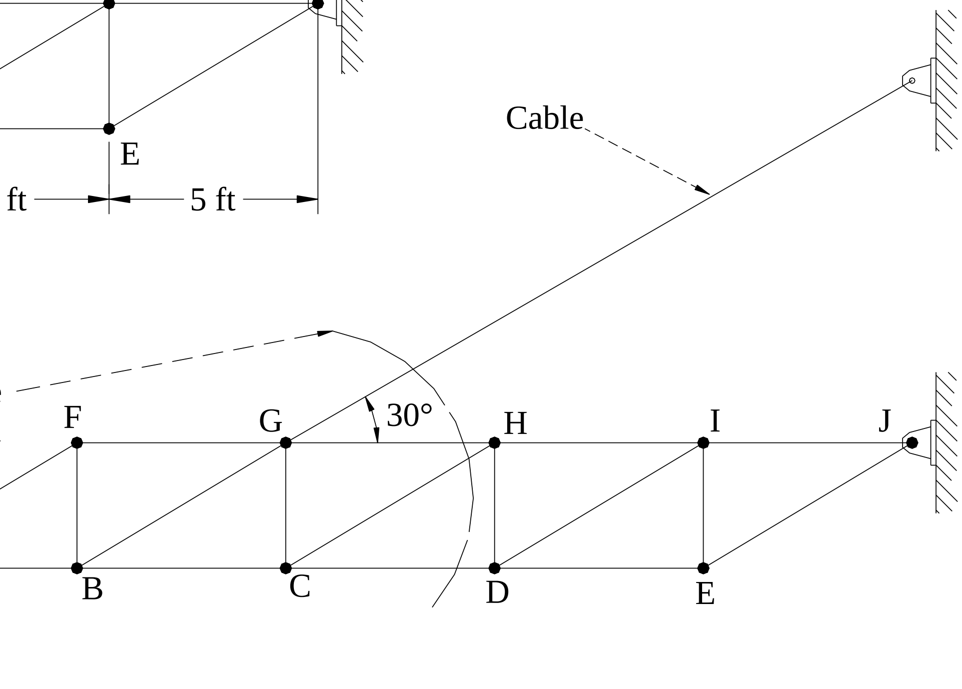
Solving gives

$F_{DK} = 2.11 \text{ kN (T)}$ ←Ans.

8. Determine the force in members GH, CD, and CH. State whether the force is tension or compression. Also, find the tension in the cable.

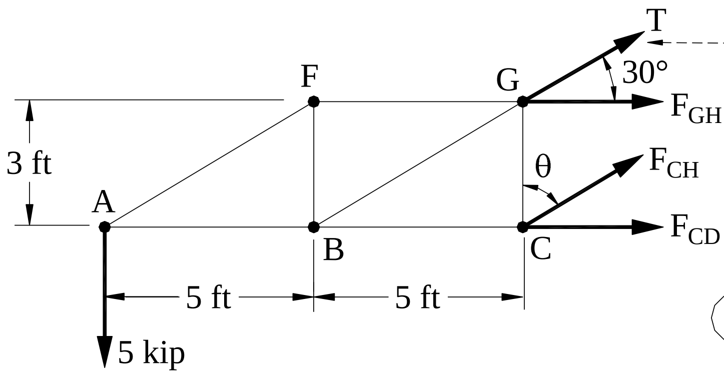


1 The section must pass through the cable. Otherwise the portion of the truss to the left of the section could not be isolated as a free body.



6.3 Trusses: Method of Sections Example 8, page 2 of 3

2 Free-body diagram of portion of truss to left of section



3 The tension in the cable is one of the unknowns.

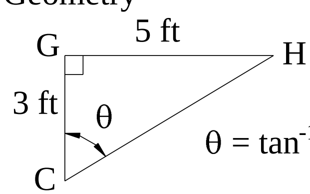
4 Equations of equilibrium for the portion of the truss:

$$\curvearrowleft + \Sigma M_C = 0: \; -T\cos 30°(3\text{ ft}) - F_{GH}(3\text{ ft}) + (5\text{ kip})(2 \times 5\text{ ft}) = 0 \qquad (1)$$

$$\curvearrowleft + \Sigma M_G = 0: \; (5\text{ kip})(2 \times 5\text{ ft}) + F_{CD}(3\text{ ft}) + F_{CH}\sin\theta\,(3\text{ ft}) = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \; -5\text{ kip} + T\sin 30° + F_{CH}\cos\theta = 0 \qquad (3)$$

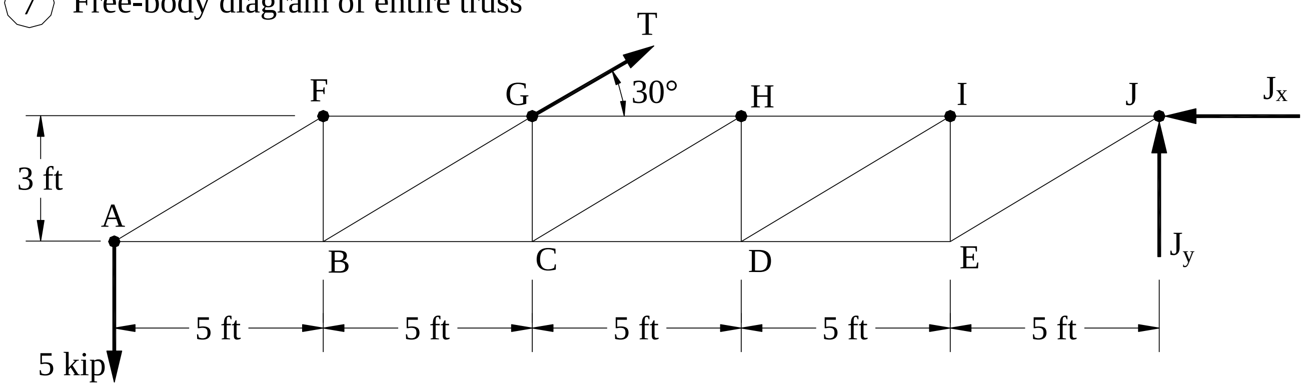
5 Geometry



$$\theta = \tan^{-1}\left(\frac{5\text{ ft}}{3\text{ ft}}\right) = 59.04°$$

6 Three equations but four unknown forces, so another equilibrium equation is needed.

(7) Free-body diagram of entire truss



(8) Equation of equilibrium for the entire truss. Only one equation is used because we need to calculate T only; the reactions at J are not needed.

$\circlearrowleft +$ $\Sigma M_J = 0$: $(5 \text{ kip})(5 \times 5 \text{ ft}) - T \sin 30°(3 \times 5 \text{ ft}) = 0$ (4)

$T = 16.67$ kip ←Ans.

(9) Substituting $\theta = 59.04°$ and $T = 16.67$ kip into Eqs. 1, 2, and 3 and solving simultaneously gives:

$F_{GH} = 2.23$ kip (T) ←Ans.

$F_{CD} = -11.11$ kip $= 11.11$ kip (C) ←Ans.

$F_{CH} = -6.48$ kip $= 6.48$ kip (C) ←Ans.