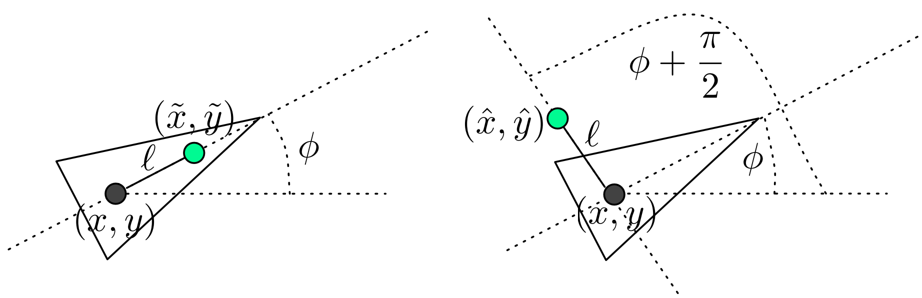
SOLUTIONS: QUIZ 7
CONTROL OF MOBILE ROBOTS

1

In order to make the unicycle

$$\dot{x} = v\cos\phi$$
$$\dot{y} = v\sin\phi$$
$$\dot{\phi} = \omega,$$

behave like a point-robot, we, in class, focused on a new point $\tilde{x} = \ell\cos\phi,\ \tilde{y} = y + \ell\sin\phi$. But, what if we instead were interested in a different point $(\hat{x}, \hat{y})$, as shown below?



The idea behind the abstraction layers is that we would like to plan as if we could control the point $(\hat{x}, \hat{y})$ directly through

$$\dot{\hat{x}} = u_1$$
$$\dot{\hat{y}} = u_2.$$

But, for this to work we need to be able to relate (u_1, u_2) to the actual control inputs of the unicycle. Which of the following expressions correctly relate (v, ω) to (u_1, u_2)?

SOLUTION

The new point is given by

$$\hat{x} = x + \ell\cos(\phi + \pi/2) = x - \ell\sin(\phi) \ \Rightarrow\ \dot{\hat{x}} = v\cos(\phi) - \ell\omega\cos(\phi)$$
$$\hat{y} = y + \ell\sin(\phi + \pi/2) = y + \ell\cos(\phi) \ \Rightarrow\ \dot{\hat{y}} = v\sin(\phi) - \ell\omega\sin(\phi).$$

Setting this expression equal to (u_1, u_2) gives

$$\begin{bmatrix} \cos\phi & -\cos\phi \\ \sin\phi & -\sin\phi \end{bmatrix} \begin{bmatrix} v \\ \ell\omega \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

But, unfortunately we note that

$$\det\left(\begin{bmatrix} \cos\phi & -\cos\phi \\ \sin\phi & -\sin\phi \end{bmatrix}\right) = -\cos\phi\sin\phi + \cos\phi\sin\phi = 0,$$

i.e., we cannot invert this matrix. As such, we cannot solve the equation for v and ω. The result from that is that the correct answer is *The point* $(\hat{x}, \hat{y})$ *is not a good point to chose since there is no way of relating* (v, ω) *to* (u_1, u_2) *directly.*

2

Consider the car-like robot model in Lecture 7.6. Assume it is driving with a constant steering angle ψ. What motion would the car execute, assuming that the translational velocity v is positive but possibly changing over time?

SOLUTION

From the equation on slide 7.6.4 we see that the car will indeed drive in a circular arc no matter what v is. The radius of the circle is moreover given by

$$\rho = \frac{\ell}{\sin(\psi)},$$

i.e., the correct answer to the question is *Drive along a circular arc with radius inversely proportional to* $\sin(\psi)$.

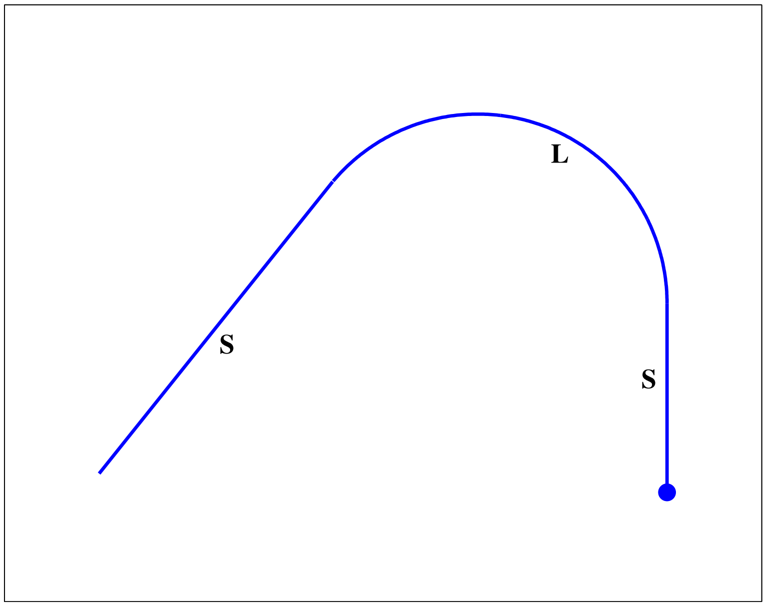
3

The Dubins vehicle model states that

$$\begin{aligned} \dot{x} &= v\cos\phi \\ \dot{y} &= v\sin\phi \\ \dot{\phi} &= \omega \\ v &= 1, \;\; \omega \in [-1, 1], \end{aligned}$$

i.e., it is a unicycle with speed $v = 1$ and where the angular velocity is bounded. We now know that this means that the Dubins vehicle can only execute maneuvers whose curvature is less than or equal to 1 (max(curvature)=$|\omega/v| = 1/1 = 1$).

When solving the problem of moving in the shortest amount of time between two points, it is possible to show that only three "modes" are used, namely go straight (S) with $\omega = 0$, turn max left (L) with $\omega = 1$, and turn max right (R), with $\omega = -1$. An example of a S-L-S maneuver is shown below, with the robot starting from the solid circle:

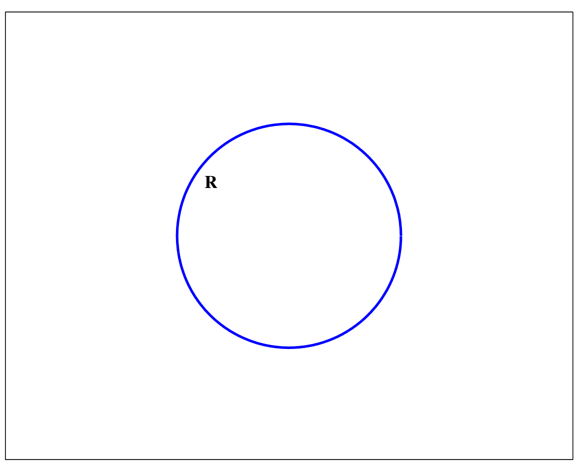


Which of the following types of maneuvers could (depending on how long each segment is) possibly move a Dubins vehicle from $(x, y, \phi) = (0, 0, 0)$ to $(x, y, \phi) = (0, 0, \pi)$?

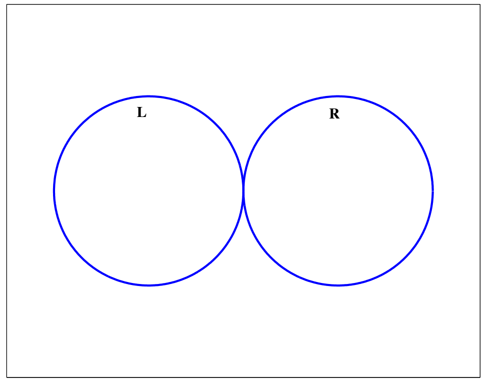
SOLUTION

The problem involves returning to the same point from which the robot started by having the robot oriented in the opposite direction as compared to the original orientation. Let's investigate the different options one by one to see if we can achieve this with the proposed control strategies (There are 2 correct solutions for this question):

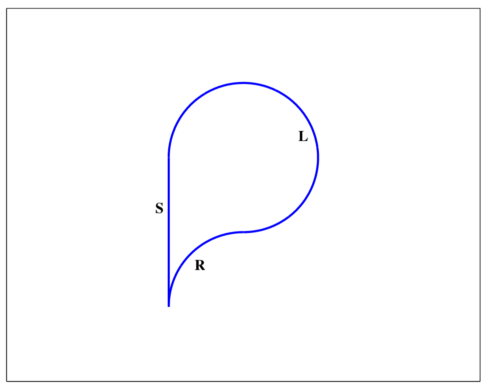
R-S: By drawing a circle centered at the origin, we see that there is simply no way of returning to the initial point unless the S-segment has length 0. But in that case, the final orientation is the same as the original orientation, and this is not the correct solution, as seen below.



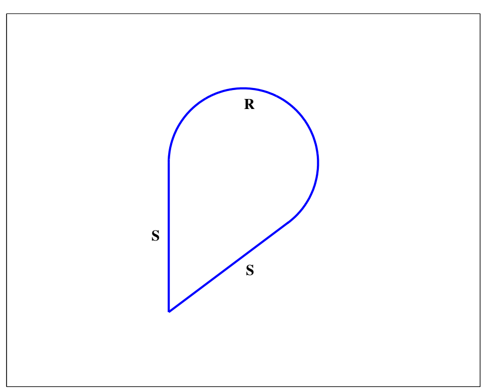
L-R: Also here we see that the only way to return to the original position is to either have one (or both) turning segments have 0 length while the other (or both) complete the circle. Also in this case, the final orientation is the same as the original orientation, i.e., this is not the correct solution.



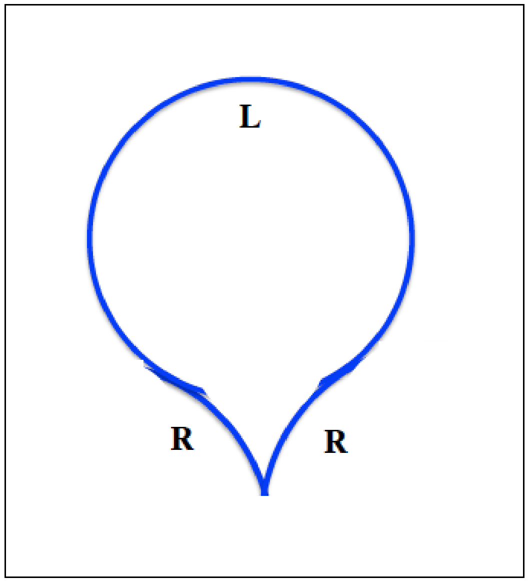
R-L-S Consider the solution shown below. Hence, this maneuver does indeed get the job done, and it is the correct solution!



S-R-S: Here it is indeed possible to return to the original position with an orientation different from the initial orientation. But, there is no way of making the final orientation opposite of the initial orientation, as seen below.



R-L-R: Consider the solution shown below. Hence, this maneuver gets the job done as well, and is hence the correct solution!



4

Why do we typically use layered architectures when designing robotic navigation systems?

SOLUTION

It allows for the details of the robot model to be abstracted away at the higher levels of the architecture. Indeed, one of the key reasons for using a layered architecture is that higher-levels should not have to worry about the details of the lower levels.

Different robot types can execute the same high-level navigation strategies. This is also correct. The same high-level layer can be used across different robotic platforms.

It makes the navigation problem easier by separating it into a planning phase and a tracking phase. This is correct and in essence it says the same thing as the previous reason.

AI-based planning tools can be more or less directly applied without having to couple them to the robot dynamics. Also true. And, again, it is basically the same reason as the two previous reasons.

They are all good reasons. This is thus the correct answer to the question!

5

Last question of the entire course – Which of the following items did not appear anywhere in this course?

SOLUTION

Banana: When discussing linear systems we observed that they are as rare as bananas are in the universe. So, yes, a banana did show up in the course.

Tortoise: To illustrate the Zeno phenomenon we considered the paradox of a tortoise racing a hare. So, the tortoise did show up in the course.

Alien: One conclusion of the course is that there are things we did not cover that we already know, but also things that we do not know yet – elegantly illustrated by an alien.

Teddy bear. There were no teddy bears in this course – this is the correct answer!

Basket ball. The bouncing ball was used as an example of a Type 2 Zeno system. And this was illustrated by a basket ball.