SOLUTIONS: QUIZ 5

CONTROL OF MOBILE ROBOTS

1

Which of the following statements is false?

SOLUTION

Let's go through the statements one by one:

The Zeno phenomenon should be avoided when designing hybrid control systems. That is certainly true since Zeno means infinitely many switches in finite time, which is bad for a number of reasons.

Even though the individual modes are stable, the composite hybrid system need not be stable. This is also true. As was seen in the lectures, it is possible to destabilize a system by switching between stable sub-systems.

Existential stability implies universal stability. Existential stability means that it is possible to construct a *single* switching sequence that renders the switched system stable. But from this it does not follow that the switched system is stable for *all* (universal stability) switching signals. As such, this statement is false, i.e., this is the correct answer to the question.

Certain types of Zeno behaviors can be removed by introducing a new mode. This is the whole point with the induced mode, i.e., the statement is correct.

Hybrid automata consist of modes, guards, transitions, and resets. Yes – the statement is true.

2

Consider the switched system

$$\dot{x} = \begin{cases} x & \text{if } x \leq 1 \\ -2x & \text{if } x > 1. \end{cases}$$

This system is Type 1 Zeno. What is the induced mode?

SOLUTION

There are two ways of approaching this question. The first is to observe that the induced mode keeps the state on the switching surface, i.e., maintains x at 1. But, for x to be constant, we need $\dot{x} = 0$, which is the induced mode.

The other option is to go through the math to derive the induced mode. From slide 5.8.4 we have that

$$\dot{x} = \frac{1}{L_{f_2}g - L_{f_1}g}(L_{f_2}gf_1 - L_{f_1}gf_2),$$

where the Lie-derivatives are

$$L_{f_i} g = \frac{\partial g}{\partial x} f_i, \quad i = 1, 2.$$

We note that $g(x) = x - 1$ and, as such

$$\frac{\partial g}{\partial x} = 1 \;\Rightarrow\; \begin{cases} L_{f_1} g = x \\ L_{f_2} g = -2x. \end{cases}$$

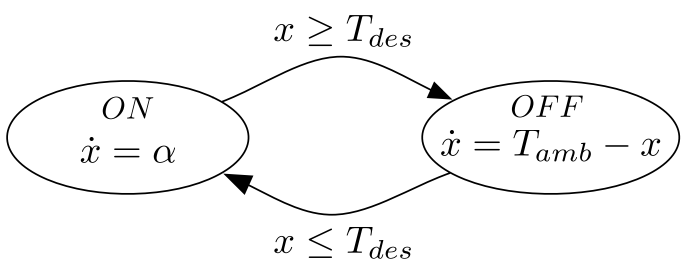
Plugging this into the induced mode formula gives

$$\dot{x} = \frac{1}{-2x - x}(-2x \cdot x - x \cdot (-2x)) = 0,$$

which again is the induced mode.

3

Below is shown a model of a thermostat in a building. In the model, x is the temperature and in the ON-mode, the thermostat is increasing the temperature at a rate of $\alpha > 0$, while in the OFF-mode, the thermostat is simply off. What this means is that the temperature will exponentially decay down to the ambient temperature T_{amb}. Moreover, the desired temperature in the thermostat is set to T_{des}, where we assume that $T_{des} > T_{amb}$.



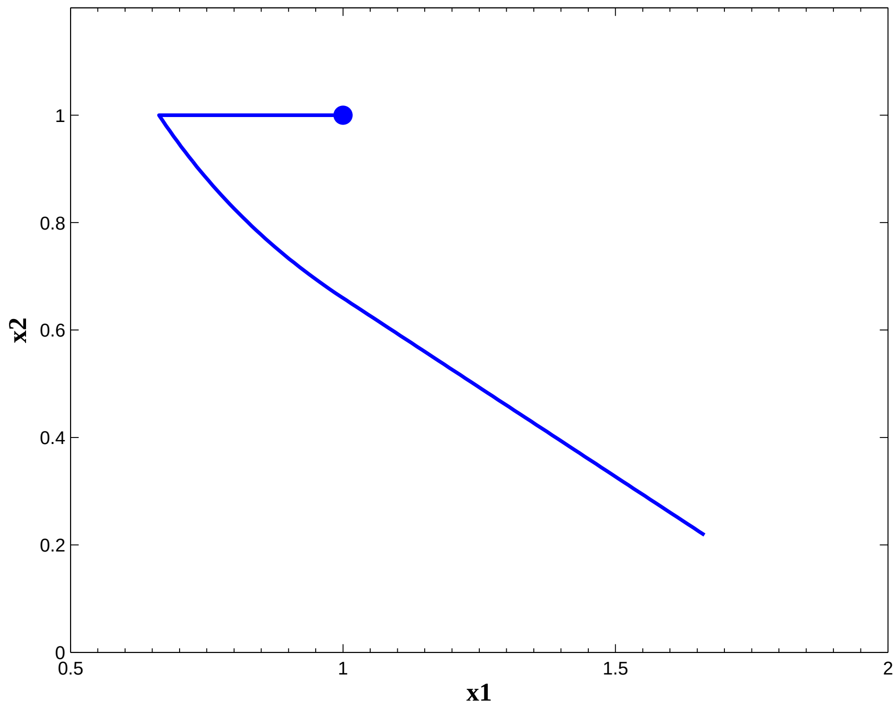
Why is the hybrid automaton a bad thermostat?

SOLUTION

When the thermostat is in the ON-mode, the temperature is increasing according to $\dot{x} = \alpha$ until $x = T_{des}$, at which point the desired temperature has been reached. At this time, the hybrid automaton switches to the OFF mode. But, since the guard condition is $x \leq T_{des}$, we have that the guard condition is satisfied immediately since $x = T_{des}$ at the time of the switch. As a consequence, the HA switches to the ON mode immediately. But then it switches back to OFF right away, and we can conclude that this is a bad thermostat since "*Once $x = T_{des}$ it will switch infinitely fast between the two modes*", which is the answer.

4

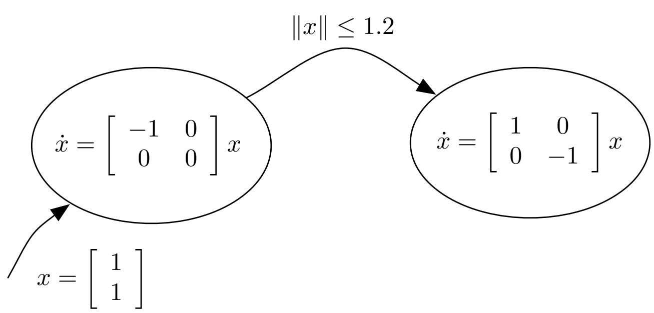
Consider the plot shown below, where the initial condition is $(x_1, x_2) = (1, 1)$ is marked by a solid circle.



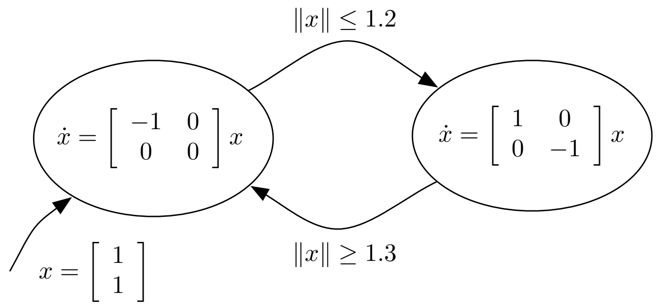
Which Hybrid Automaton was used to generate this plot?

SOLUTION

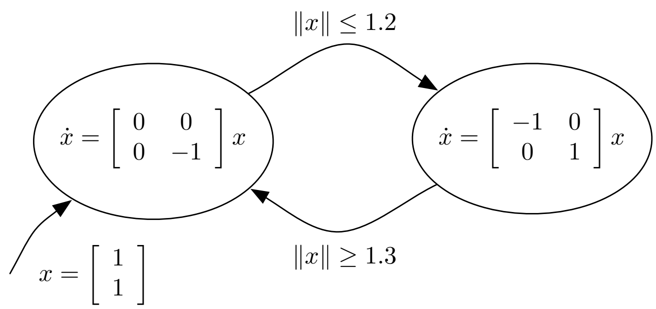
Let us go through the different options.



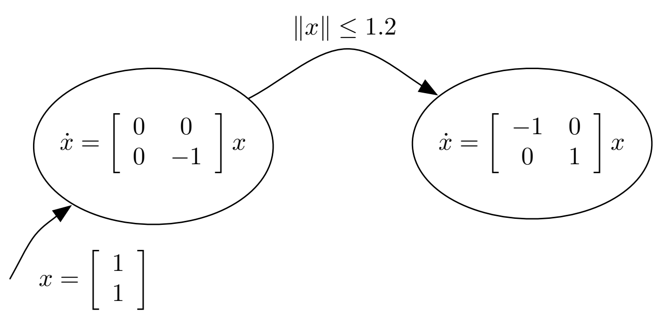
This HA starts at $(1, 1)$ which is the same as the plot. In the first mode, only the first component of x changes, through $\dot{x}_1 = -x_1$, which is consistent with the plot as well. From the plot, we moreover see that the switch occurs roughly when $(x_1, x_2) \approx (0.65, 1)$, which means that at the switch-moment $\|x\| \approx 1.2$, which is consistent with the HA. Then, the system seems to stay in that mode all the time, which is also what the plot indicates. As such, this is the correct solution!



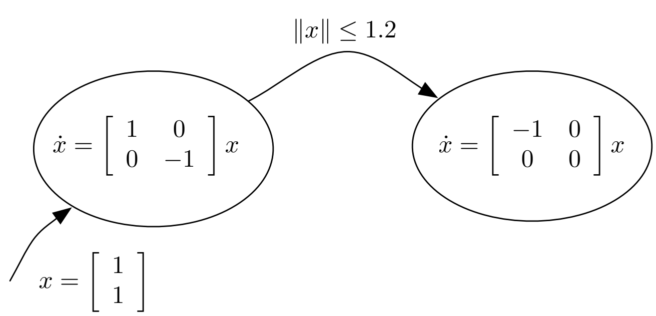
This HA starts out correctly (see previous answer), but then it is supposed to switch back to the first mode when $\|x\| = 1.3$. But, this is not what happens in the plot.



This HA starts out by decreasing x_2 instead of x_1, which is not what the plot shows. So it is not the correct HA.



Just as in the previous option, this HA starts out by decreasing x_2 instead of x_1. It is not the correct HA.



Here the initial dynamics is completely wrong – this is not the correct HA!

5

Why do we care about hybrid systems when designing robotic controllers?

SOLUTION

Let's go through the answers individually:

The environment is unknown and we cannot design a single controller that deals with all possible environmental conditions the robot may encounter. Yes, this is certainly true, which is one of the main reasons why you need to switch between different behaviors.

It makes the design easier in that the design task is broken down into building blocks. Indeed. If you design a new and improved obstacle-avoidance behavior, you only need to change that particular behavior, and not the entire navigation system.

It connects to how biological systems, e.g., animals, are thought to behave. There is strong biological evidence that such motion primitives are present in nature, i.e., this statement is also correct.

It makes the design task modular in that new functionality can be added onto already existing control structures. Absolutely! Just plop down your new behavior into the system.

They all are valid concerns. Since all options are correct, this is the answer to the question!