FOUNDATIONS FOR VIBRATING MACHINES

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ABSTRACT

The paper discusses the methods of analysis for determining the response of foundations subjected to vibratory loads. The design of a machine foundation is generally made by idealizing the foundation- soil system as spring-mass –dashpot model having one or two degrees of freedom. Most machine foundations are treated as surface footing and the soil spring and damping values are determined using the elastic-half space analog. The spring and damping values for response of embedded foundations can also be determined from the elastic half space concept as per Novak's work. The soil spring and damping values can also be obtained following the impedance –compliance function approach. The paper also presents a brief discussion of the predicted and observed response of machine foundations .

INTRODUCTION

Machine foundations require a special consideration because they transmit dynamic loads to soil in addition to static loads due to weight of foundation, machine and accessories. The dynamic load due to operation of the machine is generally small compared to the static weight of machine and the supporting foundation. In a machine foundation the dynamic load is applied repetitively over a very long period of time but its magnitude is small and therefore the soil behavior is essentially elastic, or else deformation will increase with each cycle of loading and may become unacceptable. The amplitude of vibration of a machine at its operating frequency is the most important parameter to be determined in designing a machine foundation, in addition to the natural frequency of a machine foundation soil system.

There are many types of machines that generate different periodic forces. The most important categories are:

1. Reciprocating machines: The machines that produce periodic unbalanced forces (such as steam engines) belong to this category. The operating speeds of such machines are usually less than 600r/min. For analysis of their foundations, the unbalanced forces can be considered to vary sinusoidally.
2. Impact machines: These machines produce impact loads, for instance, forging hammers. Their speeds of operation usually vary from 60 to 150 blows per minute. Their dynamic loads attain a peak in a very short interval and then practically die out.
3. Rotary machines: High-speed machines like turbogenerators or rotary compressors may have speeds of more than 3,000r/min and up to 12,000r/min.

A suitable foundation is selected, depending upon the type of machine. For compressors and reciprocating machines, a block foundation is generally provided (Fig.1a). Such a foundation consists of a pedestal resting on a footing. If two or more machines of similar type are to be installed in a shop, these can profitably be mounted on one continuous mat.

A block foundation has a large mass and, therefore, a smaller natural frequency. However, if a relatively lighter foundation is desired, a box or a caisson type foundation may be provided. (Fig.1b) The mass of the foundation is reduced and its natural frequency increases. Hammers may also be mounted on block foundations, but their details would be quite different than those for reciprocating machines.

Steam turbines have complex foundations that may consist of a system of walls columns, beams and slabs. (Fig.1c) Each element of such a foundation is relatively flexible as compared to a rigid block and box or a caisson-type foundation.

The analysis of a block foundation is relatively simple as compared to a complex foundation. There are several methods of analysis for both the block and the complex foundations. The criteria for designing machine foundations shall be discussed first followed by the methods of analysis.

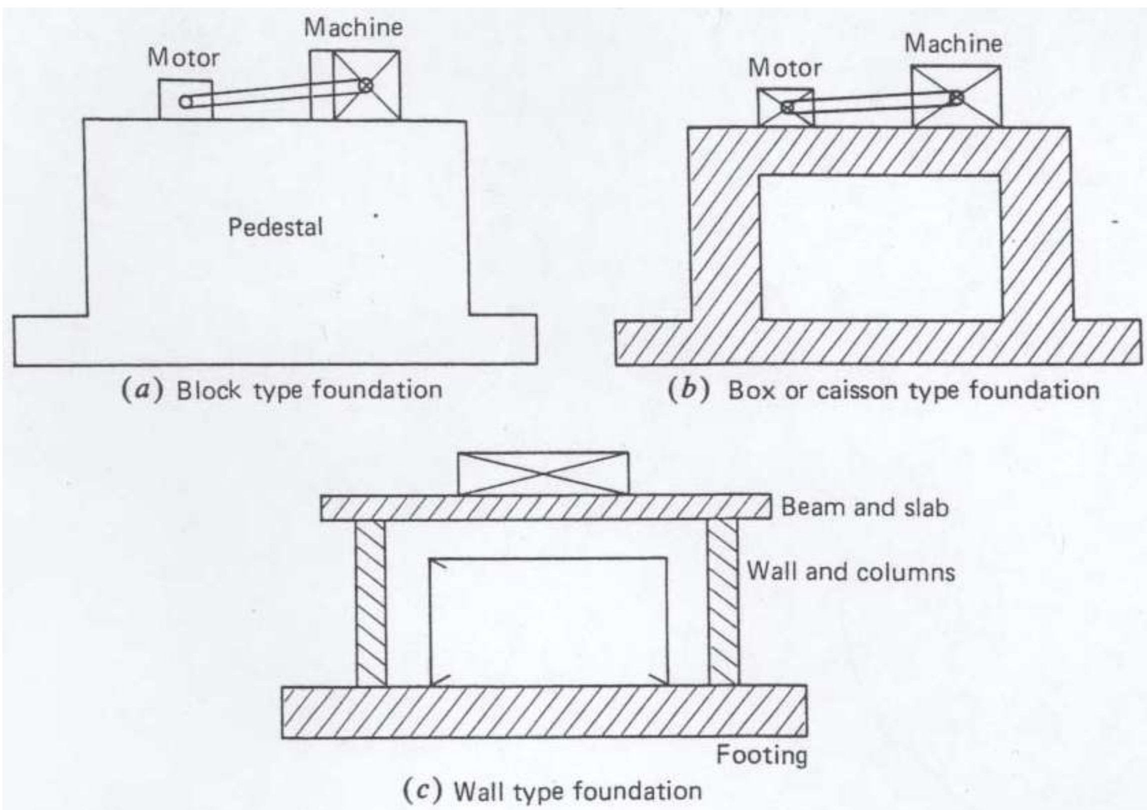

Figure 1. *Types of Machine Foundations (a) Block foundations. (b) Box or caisson foundations. (c) Complex foundations*

CRITERIA FOR DESIGN

A machine foundation should meet the following conditions for satisfactory performance:

Static loads

1. It should be safe against shear failure
2. It should not settle excessively

These requirements are similar to those for all other foundations.

Dynamic loads

1. There should be no resonance; that is, the natural frequency of the machine-foundation-soil system should not coincide with the operating frequency of the machine. In fact, a zone of resonance is generally defined and the natural frequency of the system must lie outside this zone. The foundation is high tuned when its fundamental frequency is greater than the operating speed or low tuned when its fundamental frequency is lower than the operating speed. This concept of a high or low tuned foundation is illustrated in Fig..2.
2. The amplitudes of motion at operating frequencies should not exceed the limiting amplitudes, which are generally specified by machine manufacturers. If the computed amplitude is within tolerable limits, but the computed natural frequency is close to the operating frequency, it is important that this situation be avoided.
3. The natural frequency of the foundation –soil system should not be whole number multiple of the operating frequency of the machine to avoid resonance with the higher harmonics.
4. The vibrations must not be annoying to the persons working in the shops or damaging to the other precision machines. The nature of vibrations that are perceptible, annoying, or harmful depends upon the frequency of the vibrations and the amplitude of motion.

The geometrical layout of the foundation may also be influenced by the operational requirements of the machine. The failure condition of a machine foundation is reached when its motion exceeds a limiting value which may be based on acceleration , velocity or amplitude. . Richart (1962) defined the failure criteria in terms of limiting displacement amplitudes at a given frequency. The limiting or permissible amplitudes can be established from Fig. 3 (Blake, 1964), who also introduced the concept of service factor.

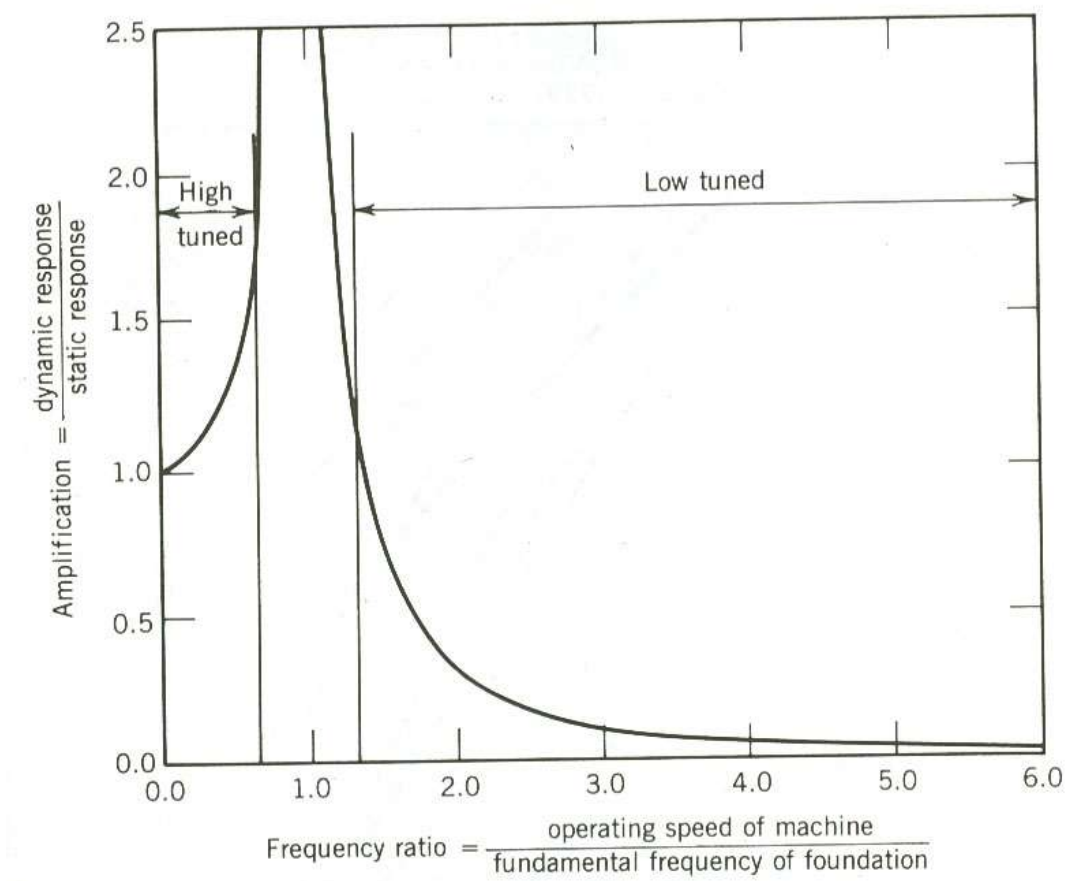

Figure2. *Tuning of a foundation*

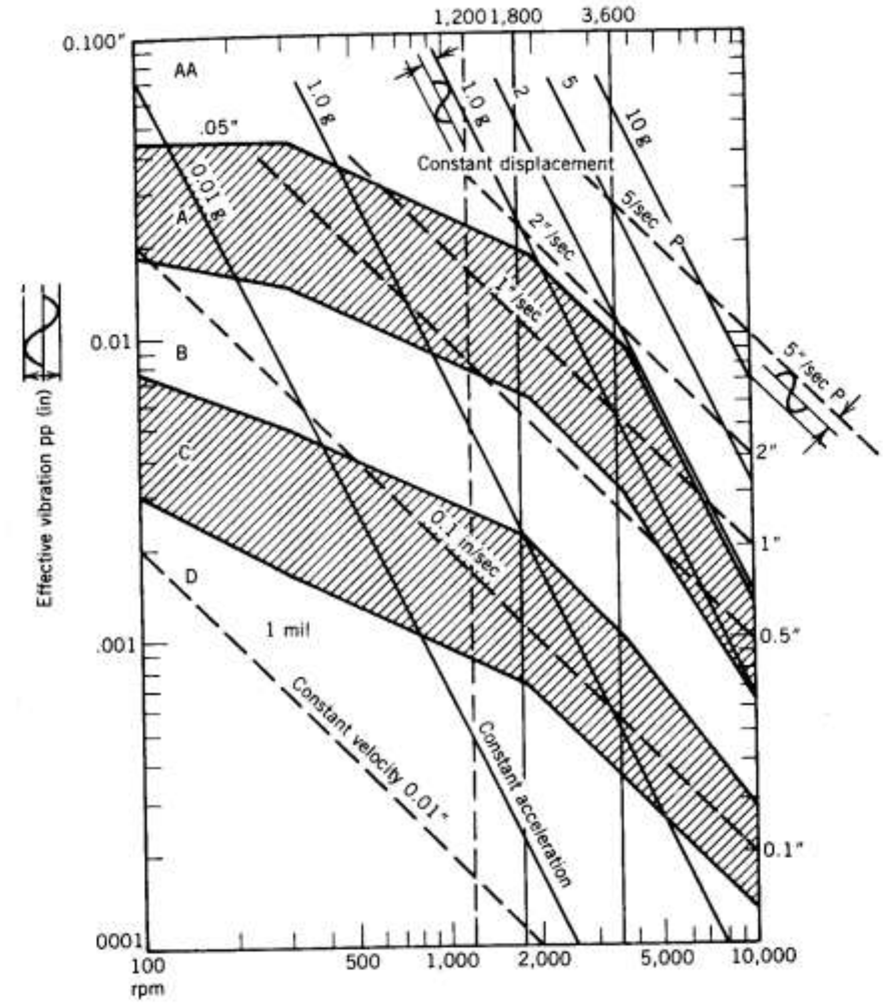

Figure 3. *Limiting amplitudes of vibrations for a particular frequency. (Blake, 1964)*

Criterion for vibration of rotating machinery. Explanation of classes :

AA Dangerous. Shut it down now to avoid danger
A Failure is near. Correct within two days to avoid breakdown.
B Faulty. correct it within 10 days to save maintenance dollars.
C Minor faults. Correction wastes dollars.
D No faults. Typical new equipment.

This is guide to aid judgment, not to replace it. Use common sense. Take account of all local circumstances. Consider: safety, labor costs, downtime costs. (after Blake, 1964.) Reproduced with permission from Hydrocarbon Processing, January 1964.

The service factor indicates the importance of a machine in an installation. Typical values of service factors are listed in Table1. Using the concept of service factor, the criteria given in Fig. 3 can be used to define vibration limits for different classes of machines. Also, with the introduction of the service factor, Fig. 3 can be used to evaluate the performance of a wide variety of machines. The concept of service factor is explained by the following examples.

A centrifuge has a 0.01 in (0.250 mm) double amplitude at 750 rpm. The value of the service factor from Table 1 is 2, and the effective vibration therefore is 2X 0.01 = 0.02 in (0.50 mm). This point falls in Class A in Fig. 3. The vibrations, therefore, are excessive, and failure is imminent unless the corrective steps are taken immediately. Another example is that of a link-suspended centrifuge operating at 1250 rpm that has0.00.30 in (0.075mm) amplitude with the basket empty. The service factor is 0.3, and the effective vibration is 0.00090 in (0.0225mm). This point falls in class C (Fig. 3) and indicates only minor fault.

General information for the operation of rotary machines is given in Table 2 (Baxter and Bernhard 1967).

These limits are based on peak-velocity criteria alone and are represented by straight lines in Fig. 3

Table 1. Service Factors [a]	
Single-stage centrifugal pump, electric motor, fan	1
Typical chemical processing equipment, noncritical	1
Turbine, turbogenerator, centrifugal compressor	1.6
Centrifuge, stiff-shaft [b]; multistage centrifugal pump	2
Miscellaneous equipment, characteristics unknown	2
Centrifuge, shaft-suspended, on shaft near basket	0.5
Centrifuge, link-suspended, slung	0.3

a Effective vibration - measured single *amplitude* vibration, in inches multiplied by the service factor. Machine tools are excluded. Values are for bolted-down equipment; when not bolted, multiply the service factor by 0.4 and use the product as the service factor. *Caution:* Vibration is measured on the bearing housing except, as stated.

b Horizontal displacement basket housing.

Table 2. General Machinery – Vibration Severity Criteria (Baxter and Bernhart, 1967)	
Horizontal Peak Velocity (in/sec)	Machine Operation
<0.005	Extremely smooth
0.005-0.010	Very smooth
0.010-0.020	Smooth
0.020-0.040	Very good
0.040-0.080	Good
0.080-0.160	Fair
0.160-0.315	Slightly rough
0.315-0.630	Rough
>0.630	Very rough

DEGREES OF FREEDOM OF A RIGID BLOCK FOUNDATION

A typical concrete block is regarded as rigid as compared to the soil over which it rests. Therefore, it may be assumed that it undergoes only rigid-body displacements and rotations. Under the action of unbalanced forces, the rigid block may thus undergo displacements and oscillations as follows (Fig. 4)

1. translation along *Z* axis
2. translation along *X* axis
3. translation along *Y* axis
4. rotation about *Z* axis
5. rotation about *X* axis
6. rotation about *Y* axis

Any rigid-body displacement of the block can be resolved into these six independent displacements. Hence, the rigid block has six degrees of freedom and six natural frequencies.

Of six types of motion, translation along the Z axis and rotation about the Z axis can occur independently of any other motion. However, translation about the X axis (or Y axis) and rotation about the Y axis (or X axis) are coupled motions. Therefore, in the analysis of a block, we have to concern ourselves with four types of motions. Two motions are independent and two are coupled. For determination of the natural frequencies, in coupled modes, the natural frequencies of the system in pure translation and pure rocking need to be determined. Also, the states of stress below the block in all four modes of vibrations are quite different. Therefore, the corresponding soil-spring constants need to be defined before any analysis of the foundations can be undertaken.

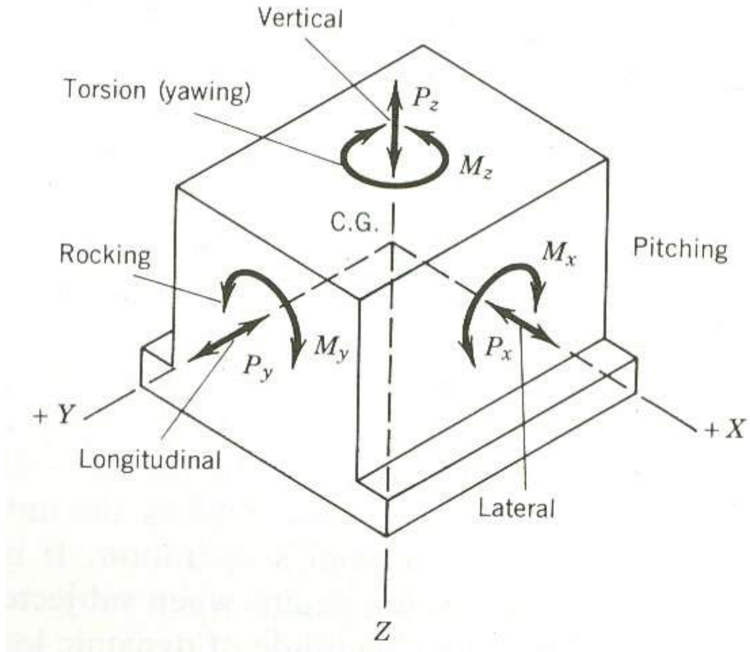

Figure 4.*Modes of vibration of a rigid block foundation*

INFORMATION NEEDED FOR DESIGN

The following information is required and must be obtained for design of a machine foundation:

1. Static weight of the machine and accessories.
2. Magnitude and characteristics of dynamic loads imposed by the machine operation and their point of application
3. The soil profile of the site and dynamic soil properties such as dynamic shear modulus and damping
4. Trial dimensions of the foundation. These are generally supplied by the manufacturer. This will give the total static weight.
5. An acceptable method of analysis i.e., a mathematical model to determine the response of the foundation-soil system
6. A criteria for adequate design

The above items are briefly discussed below:

Dynamic Loads: The information on dynamic loads and moments may be available from the manufacturer of the machine. It may be possible to determine the dynamic loads and moments for design of a machine foundation in some simple cases such as for the case of reciprocating and rotary machines.

SOIL PROFILE AND DYNAMIC SOIL PROPERTIES

Satisfactory design of a machine foundation needs information on soil profile, depth of different layers, physical properties of soil and ground water level. This information can be obtained by usual sub-surface exploration techniques. In addition, one must determine dynamic shear modulus, material damping, poisons ratio and mass density of soil for dynamic analysis of the machine foundation. Dynamic shear modulus of a soil is generally determined from laboratory or field tests. Material damping can be determined from vibration tests on soil columns in the laboratory. The values of dynamic shear modulii and damping may be estimated from empirical estimations for preliminary design purposes. Geometrical damping is estimated from elastic half-space theory and appropriate analogs. Detailed discussion of determination of dynamic soil properties and interpretation of test is beyond the scope of this paper and a reference may be made to Prakash (1981) and Prakash and Puri (1981, 1988)

TRIAL DIMENSIONS OF THE FOUNDATION

The trial dimensions of the machine foundation are selected based on the requirements of the manufacturer, the machine shop and the machine performance and experience of the designer. These trial dimensions of the foundation are only the first step in the design and may need alteration after the analysis.

METHODS OF ANALYSIS

The analysis of machine foundation is usually performed by idealizing it as a simple system as explained here. Figure 5 shows a schematic sketch of a rigid concrete block resting on the ground surface and supporting a machine. Let us assume that the operation of the machine produces a vertical unbalanced force which passes through the combined centre of gravity of the machine-foundation system. Under this condition, the foundation will vibrate only in the vertical direction about its mean position of static equilibrium. The vibration of the foundation results in transmission of waves through the soil. These waves carry energy with them. This loss of energy is termed 'geometrical damping'. The soil below the footing experiences cyclic deformations and absorbs some energy which is termed 'material damping'. The material damping is generally small compared to the geometrical damping and may be neglected in most cases. However, material damping may also become important in some cases of machine foundation vibrations.

The problem of a rigid block foundation resting on the ground surface, (Fig. 5a) may therefore be represented in a reasonable manner by a spring-mass-dashpot system shown in Fig. 5b. The spring in this figure is the equivalent soil spring which represents the elastic resistance of the soil below the base of the foundation. The dashpot represents the energy loss or the damping effect. The mass in Fig. 5b is the mass of the foundation block and the machine. If damping is neglected, a spring-mass system shown in Fig. 5c may be used to represent the problem defined in Fig. 5a. Single degree of freedom models shown in Fig. 5 b and c may in fact be used to represent the problem of machine foundation vibration in any mode of vibration if appropriate values of equivalent soil spring and damping constants are used. For coupled modes of vibration, as for combined rocking and sliding, two degree-of-freedom model is used as discussed later in the paper.

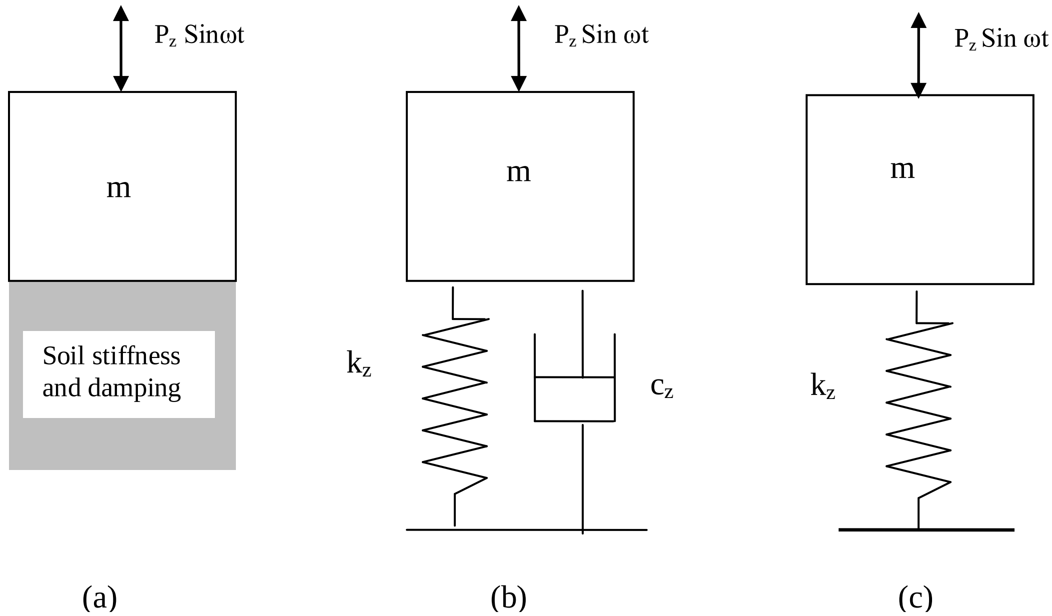

Figure 5. *Vertical Vibrations of a Machine Foundation (a) Actual case, (b) Equivalent model with damping (c) Model without damping*

All foundations in practice are placed at a certain depth below the ground surface. As a result of this embedment, the soil resistance to vibration develops not only below the base of the foundation but also along the embedded portion of the sides of the foundation. Similarly the energy loss due to radiation damping will occur not only below the foundation base but also along the sides of the foundation. The type of models shown in Fig. 5 b and c may be used to calculate the response of embedded foundations if the equivalent soil spring and damping values are suitably modified by taking into account the behavior of the soil below the base and on the sides of the foundation.

Several methods are available for analysis of vibration characteristics of machine foundations. The commonly used methods are

1 Linear elastic spring method,
2 Elastic half-space analogs method, and
3 The impedance function method.

1. The Linear Elastic Spring method (Barkan, 1962) treats the problem of foundation vibrations as spring- mass model , neglecting damping in the soil. The soil damping can be included if desired.

2. The Elastic Half Space Analogs: The elastic half space theory can be used to determine the values of equivalent soil springs and damping then make use of theory of vibrations to determine the response of the foundation. These are known as the ‘the elastic half space analogs’. They can be used for surface as well as embedded foundations. It may be mentioned here that the equivalent soil spring and damping values depend upon the ;

(i) type of soil and its properties,
(ii) geometry and layout of the foundation, and
(iii) nature of the foundation vibrations occasioned by unbalanced dynamic loads.

3. The Impedance Function Method: They also provide vales of soil spring and damping for surface and embedded foundations.

The solutions based on the elastic half space analog are commonly used for machine foundation design and are discussed first followed by the impedance function method.

Elastic-half –space -analogs

Surface Foundations

Vertical vibrations: The problem of vertical vibrations is idealized as a single degree freedom system with damping as shown in Fig. 13.15b. Hsieh (1962) and Lysmer and Richart (1966) have provided a solution .The equation of vibration is:

$$m\ddot{z} + \frac{3.4r_o^2}{(1-\nu)}\sqrt{\rho G}\,\dot{z} + \frac{4Gr_o}{(1-\nu)}s = P_z \sin\omega t \qquad 1$$

Where r_o = radius of the foundation (For non-circular foundations, appropriate equivalent radius may be used, see Eqs. 40-42).

The equivalent spring for vertical vibrations is given by

$$k_z = \frac{4Gr_o}{1-\nu} \qquad 2$$

And the damping c_z is given by

$$c_z = \frac{3.4r_o}{1-\nu}.\sqrt{\rho G} \qquad 3$$

The damping constant for vertical vibrations ξ_z is given by

$$\xi_z = \frac{0.425}{\sqrt{B_z}} \qquad 4$$

In which B_z is known as the modified mass ratio, given by

$$B_z = \frac{1-\nu}{4}.\frac{m}{\rho r_o^3} \qquad 5$$

The undamped natural frequency of vertical vibrations may now be obtained using Eqs. 6 and 7.

$$\omega_{nz} = \sqrt{\frac{k_z}{m}} \qquad (6)$$

$$f_{nz} = \frac{1}{2\pi}\sqrt{\frac{k_z}{m}} \qquad (7)$$

In which ω_{nz} = the circular natural frequency (undamped) of the soil foundation system in vertical vibration (rad/sec) and f_{nz} = natural frequency of vertical vibrations (Hz).

The amplitude of vertical vibration is obtained as:

$$A_z = \frac{P_z}{k_z\sqrt{(1-r^2)^2 + (2\xi_z r)^2}} = \frac{P_z}{k_z\left[\left(1-(\omega/\omega_{nz})^2\right)^2 + (2\xi_z \omega/\omega_{nz})^2\right]^{1/2}} \qquad (8)$$

Sliding vibrations

The equation of the analog for sliding is (Fig. 6)

$$m\ddot{x} + c_x\dot{x} + k_x x = P_z \sin\omega t \qquad (9)$$

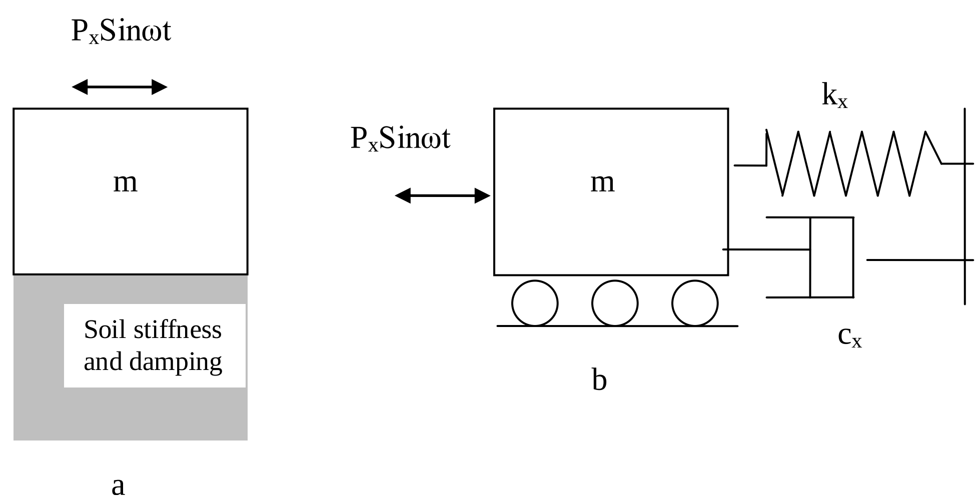

Figure 6. *Sliding Vibrations of a Rigid Block (a) Actual case (b) Equivalent model*

Hall (1967) defined the modified mass ratio for sliding as:

$$B_x = \frac{7-8\nu}{32(1-\nu)}\frac{m}{\rho r_o^3} \qquad (10)$$

where r_o = radius of the foundation .

The expressions for the equivalent spring and damping factors are as follows:
The equivalent spring

$$k_x = \frac{32(1-v)}{7-8v} G r_o \qquad 11$$

And the equivalent damping

$$c_x = \frac{18.4(1-v)}{7-8v} r_o^2 \sqrt{\rho G} \qquad 12$$

The damping ratio ξ_x is given by

$$\xi_x = \frac{c_x}{c_e} = \frac{0.2875}{\sqrt{B_x}} \qquad 13$$

The undamped natural frequency of sliding vibration may be obtained as follows:

$$\omega_{nx} = \sqrt{\frac{k_x}{m}} \qquad 14a$$

$$f_{nx} = \frac{1}{2\pi}\sqrt{\frac{k_x}{m}} \qquad 14b$$

In which ω_{nx} = the circular natural frequency (undamped) in sliding vibrations and f_{nx} = natural frequency of sliding vibrations (Hz).

The damped amplitude in sliding is obtained as:

$$A_x = \frac{P_x}{k_x \sqrt{\left(1-\left(\frac{\omega}{\omega_{nx}}\right)^2\right)^2 + \left(2\xi_x \frac{\omega}{\omega_{nx}}\right)^2}} \qquad 15$$

Rocking Vibrations: A rigid block foundation undergoing rocking vibrations due to an exciting moment $M_y \sin \omega t$ is shown in Fig. 7.

Hall (1967) proposed an equivalent mass-spring-dashpot model that can be used to determine the natural frequency and amplitude of vibration of a rigid circular footing resting on an elastic half-space and undergoing rocking vibrations (Fig.7). The equivalent model is given in equation 16

$$M_{mo}\ddot{\phi} + c_\phi \dot{\phi} + k_\phi \phi = M_y \sin \omega t \qquad 16$$

In which k_ϕ = spring constant for rocking, c_ϕ = damping constant and M_{mo} = mass moment of inertia of the foundation and machine about the axis of rotation through the base.

$$M_{mo} = M_m + mL^2 \qquad 17$$

Where M_m = mass moment of inertia of foundation and machine about an axis passing through the centroid of the system and parallel to the axis of rotation and L = the height of the centroid above the base.

The terms k_ϕ *and* c_ϕ can be obtained as follows:

$$k_\phi = \frac{8Gr_o^3}{3(1-\nu)} \qquad 18$$

And

$$c_\phi = \frac{0.8r_o^4\sqrt{G\rho}}{(1-\nu)(1+B_\phi)} \qquad 19$$

in which $r_{0\phi}$ = radius.

B_ϕ in Eq. 19 is known as the modified inertia ratio which obtained as follows:

$$B_\phi = \frac{3(1-\nu)}{8}\frac{M_{mo}}{\rho r_o^5} \qquad 20$$

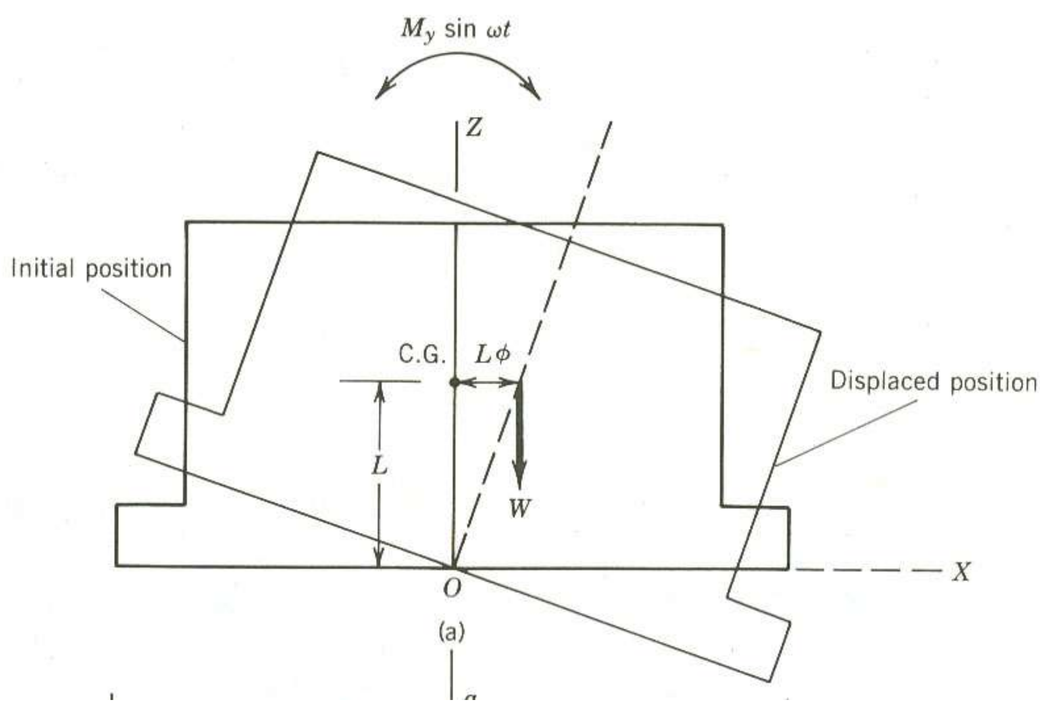

Figure 7. *Rocking vibrations of a rigid block under excitation due to an applied moment*

The damping factor ξ_ϕ is given by

$$\xi_\phi = \frac{c_\phi}{c_{\phi c}} = \frac{0.15}{(1+B_\phi)\sqrt{B_\phi}} \qquad 21$$

The undamped natural frequency of rocking

$$\omega_{n\phi} = \sqrt{\frac{k_\phi}{M_{mo}}}\ rad/\sec \qquad 22$$

Damped amplitude of rocking vibrations A_ϕ is given by Eq. 23

$$A_\phi = \frac{M_y}{k_\phi \sqrt{\left(1-\left(\frac{\omega}{\omega_{n\phi}}\right)^2\right)^2 + \left(2\xi_\phi \frac{\omega}{\omega_{n\phi}}\right)^2}} \qquad 23$$

Torsional vibrations: A block foundation undergoing torsional vibrations is shown in Fig.8. Non-uniform shearing resistance is mobilized during such vibrations. The analog solution for torsional vibrations is provided by Richart et al, (1970).

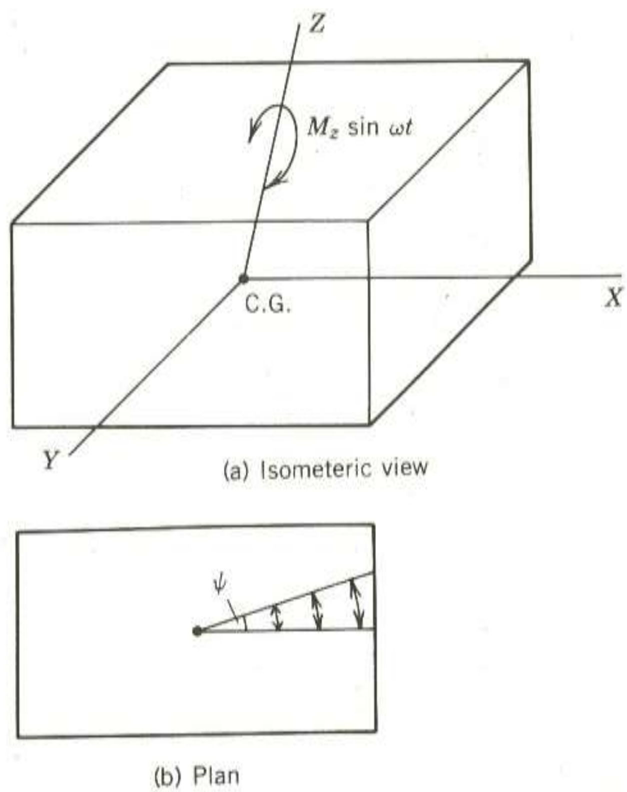

Figure 8. *Torsional vibrations of rigid block: (a) Block subjected to horizontal moment. (b) Development of nonuniform shear below the base*

The equation of motion is

$$M_{mz}\ddot{\psi} + C_\psi \dot{\psi} + k_\psi \psi = M_z e^{i\omega t} \qquad 24$$

In which M_{mz} = mass moment of inertia of the machine and foundation about the vertical axis of rotation (polar mass moment of inertia). The spring constant k_ψ and the damping constant c_ψ are given by (Richart and Whitman, 1967):

$$k_\psi = \frac{16}{3} G r_o^3 \qquad 25$$

$$c_\psi = \frac{1.6 r_o^4 \sqrt{G\rho}}{1 + B_\psi} \qquad 26$$

where $r_o (r_{o\psi})$ = equivalent radius..

The undamped natural frequency $\omega_{n\psi}$ of the torsional vibrations is given by

$$\omega_{n\psi} = \sqrt{\frac{k_\psi}{M_{mz}}}\, rad/\sec \qquad 27$$

The amplitude of vibration A_ψ is given by

$$A_\psi = \frac{M_z}{k_\psi \sqrt{\left(1 - \left(\frac{\omega}{\omega_{n\psi}}\right)^2\right)^2 + \left(2\xi\psi \frac{\omega}{\omega_{n\psi}}\right)^2}} \qquad 28$$

In which the damping ratio ξ_ψ is given by

$$\xi_\psi = \frac{0.5}{1 + 2B_\psi} \qquad 29$$

The modified inertia ratio B_ψ is given by

$$B_\psi = \frac{M_{mz}}{\rho r_o^5} \qquad 30$$

Combined rocking and sliding: The problem of combined rocking and sliding is shown schematically in Fig. 9. The equations of motion are written as:

$$m\ddot{x} + c_x\dot{x} + k_x x - Lc_x\dot{\phi} - Lk_x\phi = P_x e^{i\omega t} \qquad 31$$

$$M_m\ddot{\phi} + (C_\phi + L^2C_x)\dot{\phi} + (k_\phi + L^2k_x)\phi - Lc_x\dot{x} - Lk_x x = M_y e^{i\omega t} \qquad 32$$

The undamped natural frequencies for this case can be obtained from Eq. 33.

$$\omega_n^4 - \left(\frac{\omega_{nx}^2 + \omega_{n\phi}^2}{\gamma}\right)\omega_n^2 + \frac{\omega_{nx}^2.\omega_{n\phi}^2}{\gamma} = 0 \qquad 33$$

In which

$$\gamma = \frac{M_m}{M_{mo}} \qquad 34$$

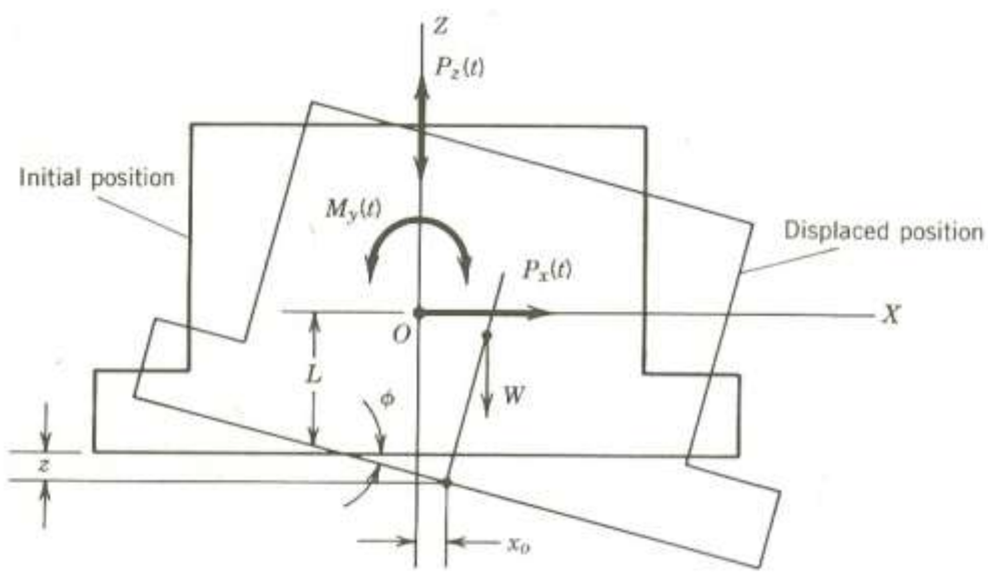

Figure 9. *Block subjected to the action of simultaneous vertical $P_z(t)$, horizontal $P_x(t)$ forces and moment $M_y(t)$*

The damping in rocking and sliding modes will be different. Prakash and Puri (1988) developed equations for determination of vibration amplitudes for this case. Damped amplitudes of rocking and sliding occasioned by an exciting moment M_y can be obtained as follows:

$$A_x = \frac{M_y}{M_m}\frac{\left[(\omega_{nx}^2)^2 + (2\xi_x\omega.\omega_{nx})^2\right]^{1/2}.L}{\Delta(\omega^2)} \qquad 356$$

$$A_\psi = \frac{M_y}{M_m} \frac{\left[(\omega_{nx}^2 - \omega^2)^2 + (2\xi_x \omega_{nx}\omega)^2\right]^{1/2}}{\Delta(\omega^2)} \qquad 36$$

The value of $\Delta(\omega^2)$ is obtained from Eq. 38

$$\Delta(\omega^2) = \left[\left(\omega^4 - \omega^2\left\{\frac{\omega_{n\phi}^2 + \omega_{nx}^2}{\gamma} - \frac{4\xi_x\xi_\phi\omega_{nx}\omega_{n\phi}}{\gamma}\right\} + \frac{\omega_{nx}^2\omega_{n\phi}^2}{\gamma}\right)^2 + 4\left\{\xi_x\frac{\omega_{nx}\omega}{\gamma}(\omega_{n\phi}^2 - \omega^2) + \frac{\xi_\phi\omega_{n\phi}\omega}{\gamma}(\omega_{nx}^2 - \omega^2)\right\}^2\right]^{1/2} \qquad 37$$

Damped amplitudes of rocking and sliding occasioned by a horizontal force P_x are given by Eqs.38 and 39

$$A_x = \frac{P_x}{mM_m} \frac{\left[(-M_m\omega^2 + k_\phi + L^2k_x)^2 + 4\omega^2(\xi_\phi\sqrt{k_\phi M_{mo}} + L^2\xi_x\sqrt{k_x m})^2\right]^{1/2}}{\Delta(\omega^2)} \qquad 38$$

And

$$A_\phi = \frac{P_x L}{M_m} \frac{\omega_{nx}(\omega_{nx}^2 + 4\xi_x\omega^2)^{1/2}}{\Delta\omega^2} \qquad 39$$

In case the footing is subjected to the action of a moment and a horizontal force, the resulting amplitudes of sliding and rocking may be obtained by adding the corresponding solutions from Eqs.35, 36, 38 and 39.

Effect of shape of the foundation on its response: The solutions from the elastic half-space theory were developed for a rigid circular footing. The vibratory response for non-circular foundations may be obtained using the concept of equivalent circular footing. The equivalent radius of the foundation for different modes of vibration is not the same. For vertical and sliding vibrations:

$$r_o = r_{oz} = r_{ox} = \left(\frac{ab}{\pi}\right)^{1/2} \qquad 40$$

For rocking vibrations

$$r_{o\phi} = r_o = \left(\frac{ba^3}{3\pi}\right)^{1/4} \qquad 41$$

For torsional vibrations

$$r_{o\psi} = r_o = \left(\frac{ab\,(a^2 + b^2)}{6\pi} \right)^{1/4} \qquad 42$$

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Foundations on elastic layer: The elastic half-space solution is based on the assumption of a homogenous soil deposit. In practice soils are layered media with each layer having different characteristics. An underlying rock below a soil layer may cause large magnification of amplitude of vibration because of its ability to reflect wave energy back into the soil supporting the foundation. Special care should be taken during design to overcome this effect.

Embedded Foundations

The embedment of the foundation results in an increased contact between the soil and the vertical faces of the foundation. This results in increased mobilization of soil reactions which now develop not only below the base of the foundation but also along the vertical sides of the foundation in contact with the soil. The overall stiffness offered by the soil therefore increases. Similarly, more energy is carried away by the waves which now originate not only from the base of the foundation but also from the vertical faces of the foundation in contact with the soil. This results in an increased geometrical damping. The elastic half-space method for calculating the response of embedded foundations was developed by Novak and Beredugo (1971, 1972), Beredugo (1976), Novak and Beredugo (1972) and Novak and Sachs (1973) by extending the earlier solution of Baranov (1967). The solution is based upon the following assumptions:

1) The footing is rigid.
2) The footing is cylindrical.
3) The base of the footing rests on the surface of a semi-infinite elastic half-space.
4) The soil reactions at the base are independent of the depth of embedment.
5) The soil reactions on the side are produced by an independent elastic layer lying above the level of the footing base.
6) The bond between the sides of the footing and the soil is perfect.

Based on the above assumptions, the expressions for equivalent spring and damping values for different modes of vibrations were obtained. The soil properties below the base of foundation were defined in terms of the shear modulus G, Poisson's ratio v and the mass density of the soil ρ. The properties of the soil on the sides of the foundation were similarly defined in terms of shear modulus G_s, the Poisson's ratio v_s and the mass density ρ_s. The values of equivalent spring and damping for vertical, sliding, rocking and torsional modes of vibrations were then obtained. The values of spring and damping were found to be frequency dependent. However, it was found that within the range of practical interest, the equivalent spring and damping may be assumed to be frequency independent. This range was defined using a dimensional frequency ratio a_o. The dimensional frequency ratio is defined as:

$$a_o = \frac{\omega r_o}{v_s} \qquad 43$$

in which ω = operating speed of the machine in rad/sec.

The values of equivalent frequency-independent spring and damping for the embedded foundation for the vertical, sliding, rocking and torsional modes are given in the Tables 3 and 4. The vibratory response of the foundation may then be calculated using the appropriate equations as for the elastic half-space analog for the surface foundations after replacing the spring stiffness and damping values with the corresponding values for the embedded foundations.

The response of a foundation undergoing coupled rocking and sliding vibrations may similarly be calculated. However, some cross-coupling stiffness and damping terms appear in the analysis of embedded foundations according to the elastic half-space method (Beredugo and Novak, 1972). The necessary equations for calculating the stiffness, damping, natural frequencies and amplitude of vibrations are summarized in Table 5.

For a given size and geometry of the foundation, and the soil properties, the stiffness and damping values for an embedded foundation are much higher than those for a surface foundation. The natural frequency of an embedded foundation will be higher and its amplitude of vibration will be smaller compared to a foundation resting on the surface. Increasing the depth of embedment may be a very effective way of reducing the vibration amplitudes. The beneficial effects of embedment, however, depend on the quality of contact between the embedded sides of the foundation and the soil. The quality of contact between the sides of the foundation and the soil depends upon the nature of the soil, the method of soil placement and its compaction, and the temperature. Reduced values of soil parameters should be used for the soil on the sides of the foundation if any 'gap' is likely to develop between the foundation sides and the soil, especially near the ground surface.

Impedance Function Method

(Surface and Embedded Foundations)

The dynamic response of a foundation may be calculated by the impedance function method. (Gazettas 1983, 1991a, b, Dobry and Gazettas 1985) This method is briefly discussed here. The geometry of rigid massless foundation considered by Gazettas (1991b) is shown in Fig.10a for a surface foundation in Fig.10b for an embedded foundation. The response of this foundation due to a sinusoidal excitation can be obtained following theory of vibration after the appropriate dynamic impedance functions $S(\omega)$ for the frequencies of interest have been determined.

The dynamic impedance is a function of the foundation soil system and the nature and the type of exciting loads and moments. For each particular case, of harmonic excitation, the dynamic impedance is defined as the ratio between force (or moment) R and the resulting steady-state displacement (or rotations) U at the centriod of the base of the massless foundation. For example, the vertical impedance is defines by

$$S_z = \frac{R_z(t)}{U_z(t)} \qquad 44$$

In which $R_z(t) = R_z \exp(i\omega t)$ and is the harmonic vertical force; and $U_z(t) = U_z \exp(i\omega t)$ harmonic vertical displacement of the soil-foundation interface. The quantity R_z is the total dynamic soil reaction against the foundation and includes normal traction below the base and frictional resistance along the vertical sides of the foundation.

The following impedances may similarly be defined: S_y = lateral sliding or swaying impedance (force-displacement ratio), for horizontal motion in the y- direction; S_x = longitudinal swaying or sliding impedance (force-displacement ratio), for horizontal motion along x-direction; S_{rx} = rocking impedance (moment-rotation ratio), for rotational motion about the centroidal x-axis of the foundation base.

Table 3. *Value of equivalent spring and damping constants for embedded foundations (Beredugo and Novak 1972, Novak and Beredugo 1972, Novak and Sachs 1973)*

Equivalent spring	**Equivalent Damping constant**	**Damping ratio**	
$k_{ze} = Gr_o\left[\bar{C}_1 + \frac{G_s}{G}\frac{h}{r_o}\bar{S}_1\right]$	$c_{ze} = r_o^2\sqrt{\rho G}\left(\bar{C}_2 + \bar{S}_2\frac{h}{r_o}\sqrt{\frac{\rho_s}{\rho}\frac{G_z}{G}}\right)$	$\xi_{ze} = \frac{c_{ze}}{2m\omega_{nze}}$	The values of frequency independent parameters $\bar{c}$'s for the elastic space are given in Table 4.
$k_{xe} = Gr_o\left[\bar{C}_{x1} + \frac{G_s}{G}\frac{h}{r_o}\bar{S}_{x1}\right]$	$c_{xe} = \sqrt{G\rho}r_o^2\left(\bar{C}_{x2} + \frac{h}{r_o}\sqrt{\frac{\rho_z}{\rho}\frac{G_z}{G}}\bar{S}_{x2}\right)$	$\xi_{xe} = \frac{c_x}{2m\omega_{nxe}}$	The values of frequency independent parameters $\bar{s}$'s for the elastic space are given in Table 4.
$k_{\phi e} = Gr_o^3\left\{\bar{C}_{\phi 1} + \frac{G_s}{G}\left(\frac{h}{r_o}\right)\left(\bar{S}_{\phi 1} + \frac{h^2}{3r_o^2}\bar{S}_{x1}\right)\right\}$	$c_{\phi e} = \sqrt{\rho G}r_o^4\left\{\bar{C}_{\phi 2} + \frac{G_s}{G}\frac{h}{r_o}\left(\bar{S}_{\phi 2} + \frac{1}{3}\frac{h^2}{r_o^2}\bar{S}_{x2}\right)\right\}$	$\xi_{\phi e} = \frac{c_\phi}{2M_{mo}m\omega_{n\phi e}}$	r_o and h refer to radius and depth of embedment of the foundation respectively
$k_{\psi e} = Gr_o^3\left(\bar{C}_{\psi 1} + \left(\frac{G_s}{G}\frac{h}{r_o}\right)\bar{S}_{\psi 1}\right)$	$c_{\psi e} = r_o^4\sqrt{\rho G}\left[\bar{C}_{\psi 2} + \bar{S}_{\psi 2}\frac{h}{r_o}\sqrt{\frac{\rho_s}{\rho}\frac{G_s}{G}}\right]$	$\xi_{\psi e} = \frac{c_\psi}{2M_{mz}m\omega_{n\psi e}}$	

Mode of Vibration
Vertical
Sliding
Rocking
Torsional or Yawing

Table4. *Values of elastic half-space and side layer parameters for embedded foundations (Beredugo and Novak 1972, Novak and Beredugo 1972, Novak and Sachs 1973)*

Mode of vibration	**Poisson's ratio** v	**Elastic half-space**		**Side layer**	
		Frequency-independent constant parameter	Validity range	Frequency-independent constant parameter	Validity range
Vertical	0.0	$\bar{C}_1 = 3.90$ $\bar{C}_2 = 3.50$	$0 \le a_0 \le 1.5$ (for all values of v)	$\bar{S}_1 = 2.70$ $\bar{S}_2 = 6.7$ (for all values of v)	$0 \le a_0 \le 1.5$ (for all values of v)
	0.25	$\bar{C}_1 = 5.20$ $\bar{C}_2 = 5.00$			
	0.5	$\bar{C}_1 = 7.50$ $\bar{C}_2 = 6.80$			
Sliding	0	$\bar{C}_{x1} = 4.30$ $\bar{C}_{x2} = 2.70$	$0 \le a_0 \le 2.0$	$\bar{S}_{x1} = 3.60$ $\bar{S}_{x2} = 8.20$	$0 \le a_0 \le 1.5$
	0.25			$\bar{S}_{x1} = 4.00$ $\bar{S}_{x2} = 9.10$	$0 \le a_0 \le 2.0$ $0 \le a_0 \le 1.5$
	0.4			$\bar{S}_{x1} = 4.10$ $\bar{S}_{x2} = 10.60$	$0 \le a_0 \le 2.0$ $0 \le a_0 \le 1.5$
	0.5	$\bar{C}_{x1} = 5.10$ $\bar{C}_{x2} = 0.43$	$0 \le a_0 \le 2.0$		
Rocking	0	$\bar{C}_{\phi 1} = 2.50$ $\bar{C}_{\phi 2} = 0.43$	$0 \le a_0 \le 1.0$	$\bar{S}_{\phi 1} = 2.50$ $\bar{S}_{\phi 2} = 1.80$ (for any value of v)	$0 \le a_0 \le 1.5$
Torsional or yawing	Any value	$\bar{C}_{\psi 1} = 4.3$	$0 \le a_0 \le 2.0$	$\bar{S}_{\psi 1} = 12.4$ $\bar{S}_{\psi 1} = 10.2$	$0 \le a_0 \le 2.0$ $0.2 \le a_0 \le 2.0$
		$\bar{C}_{\psi 2} = 0.7$		$\bar{S}_{\psi 2} = 2.0$ $\bar{S}_{\psi 2} = 5.4$	$0 \le a_0 \le 2.0$ $0.2 \le a_0 \le 2.0$

Table5 *Computation of response of an embedded foundation by elastic half-space method for coupled rocking and sliding (Beredugo and Novak 1972)*

Item	Equation
Stiffness in sliding	$k_{xe} = Gr_o\left(\bar{C}_{x1} + \frac{G_s}{G}\frac{h}{r_o}\bar{S}_{x1}\right)$
Stiffness in rocking	$k_{\phi e} = Gr_o^3\left\{\bar{C}_{\phi 1} + \left(\frac{L}{r_o}\right)^2\bar{C}_{x1} + \frac{G_s}{G}\left(\frac{h}{r_o}\right)\bar{S}_{\phi 1} + \frac{G_s}{G}\left(\frac{h}{r_o}\right)\times\left[\left(\frac{h^2}{3r_o^2} + \frac{L^2}{r_o^2} - \frac{hL}{r_o^2}\right)\bar{S}_{x1}\right]\right\}$
Cross-coupling stiffness	$k_{x\phi e} = -Gr_o\left\{L\bar{C}_{x1} + \frac{G_s}{G}\left(\frac{h}{r_o}\right)\left(L - \frac{h}{2}\right)\bar{S}_{x1}\right\}$
Damping constant in sliding	$c_{xe} = \sqrt{\rho G r_o^2}\left(\bar{C}_{x2} + \left(\frac{h}{r_o}\right)\sqrt{\frac{\rho_s}{\rho}\frac{G_s}{G}}\bar{S}_{x2}\right)$
Damping constant in rocking	$c_{\phi e} = \sqrt{\rho G r_o^4}\left\{\bar{C}_{\phi 2} + \left(\frac{L}{r_o}\right)^2\bar{C}_{x2} + \left(\frac{h}{r_o}\right) + \left(\frac{h}{r_o}\right)\sqrt{\frac{\rho_s}{\rho}\frac{G_s}{G}}\times\left[\bar{S}_{\phi 2} + \left(\frac{h^2}{3r_o^2} + \frac{L^2}{r_o^2} - \frac{hL}{r_o^2}\right)\bar{S}_{x2}\right]\right\}$
Cross-coupling damping	$c_{x\phi e} = -\sqrt{\rho G r_o^2}\left[L\bar{C}_{x2} + \left(\frac{h}{r_o}\right)\sqrt{\frac{\rho_s}{\rho}\frac{G_s}{G}}\left(L - \frac{h}{2}\right)\bar{S}_{x2}\right]$
Frequency equation	$(k_{xe} - m\omega_n^2)(k_{\phi e} - M_m\omega_n^2) - k_{x\phi e}^2 = 0$
Amplitude in sliding (damped)	$A_{xe} = P_x\sqrt{\frac{\alpha_1^2 + \alpha_2^2}{\varepsilon_1^2 + \varepsilon_2^2}}$
Amplitude in rocking (damped)	$A_{\phi e} = M_y\sqrt{\frac{\beta_1^2 + \beta_2^2}{\varepsilon_1^2 + \varepsilon_2^2}}$
Various terms in equations for A_{xe} *and* $A_{\phi e}$	$\alpha_1 = k_{\phi e} - M_m\omega^2 - \left(\frac{M_y}{P_x}\right)k_{x\phi e}$ $\alpha_2 = \left(c_{\phi e} - \frac{M_y}{P_x}c_{x\phi e}\right)\omega$ $\beta_1 = k_{xe} - m\omega^2 - \frac{P_x}{M_y}k_{x\phi e}$ $\beta_2 = \left(c_{xe} - \frac{P_x}{M_y}c_{x\phi e}\right)\omega$ $\varepsilon_1 = mM_m\omega^4 - (mk_{\phi e} + M_m k_{xe} + c_{xe}c_{\phi e} - c_{x\phi e}^2)\omega^2 + (k_{xe}k_{\phi e} - k_{x\phi e}^2)$ $\varepsilon_2 = -(mc_{\phi e} + M_m c_{xe})\omega^3 + (c_{xe}k_{\phi e} + c_{\phi e}k_{xe} - 2c_{x\phi e}k_{x\phi e})\omega$

The values of parameters $\bar{C}_{x1}, \bar{C}_{x2}, \bar{C}_{\phi 1}, \bar{C}_{\phi 2}, \bar{S}_{x1}, \bar{S}_{x2}, \bar{S}_{\phi 1},$ *and* $\bar{S}_{\phi 2}$ are given in Table 4.

L is the height of the centre of gravity above the base.

The horizontal force P_x and the moment M_y act at the centre of gravity of the foundation.

The equations given in this table are used for coupled rocking and sliding of embedded foundations only.

S_{ry} = rocking impedance (moment-rotation ratio), for rotational motion about the short centroidal axis (y) of the foundation basement; and S_I = torsional impedance (moment-rotation ratio), for rotational oscillation about the vertical axis (z).

In case of an embedded foundations, horizontal forces along principal axes induce rotational (in addition to translational) oscillations; hence two more 'cross-coupling' horizontal-rocking impedances exist S_{x-ry} *and* S_{y-rx}. They are negligible for surface and shallow foundations, but their effects may become significant as depth of embedment increases.

Material and radiation damping are present in all modes of vibration. As a result *R* is generally out of phase with *U*. It has become traditional to introduce complex notation and to express each of the impedances in the form

$$S = \bar{K} + i\omega c \qquad 45$$

in which both $\bar{K}$ and *C* are functions of the frequency ω. The real component, $\bar{K}$ is the dynamic stiffness, and reflects the stiffness and inertia of the supporting soil. Its dependence on frequency is attributed solely to the influence that frequency exerts on inertia, since soil properties are practically frequency independent. The imaginary component, ωC, is the product of the (circular) frequency ω times the dashpot coefficient, *C*. *C* is the radiation and material damping generated in the system (due to energy carried by waves spreading away from the foundation and energy dissipated in the soil by hysteric action, respectively).

Equation 45 suggests that for each mode of oscillation an analogy can be made between the actual foundation-soil system and the system that's consists of the same foundation, but is supported on a spring and dashpot with characteristic moduli equal to $\bar{K}$ *and* C, respectively.

Gazettas (1991a, b) presented a set of tables and figures for determination of dynamic stiffness and damping for various modes of vibration of a rigid foundation as shown in Tables 6 and 7 and Figs. 11 and 12.

Table 6 and Fig 11 contain the dynamic stiffness (springs), $\bar{K} = \bar{K}(\omega)$ for surface foundations. Each stiffness is expressed as a product of the static stiffness, *K*, times the dynamic stiffness coefficient $k = k(\omega)$.

$$\bar{K}(\omega) = K.k(\omega) \qquad 46$$

Table 7 and Fig. 12 similarly give the information for an embedded foundation. Tables 6 and 7 and Figs. 11 and 12 contain the radiation damping (dashpot) coefficients, $C = C(\omega)$. These coefficients do not include the soil hysteric damping β. To incorporate such damping, one may simply add the corresponding material dashpot constant $2\bar{K}\beta/\omega$ to the radiation *C* value.

$$total\ \ C = radiation\ C + \frac{2\bar{K}}{\omega}\beta \qquad 47$$

Gazettas (1991a, b) has also illustrated the procedure for calculating the response of the foundation using the impedance method. The solutions have also been developed for a rigid footing resting or partly embedded into a stratum (Gazettas, 1991a).

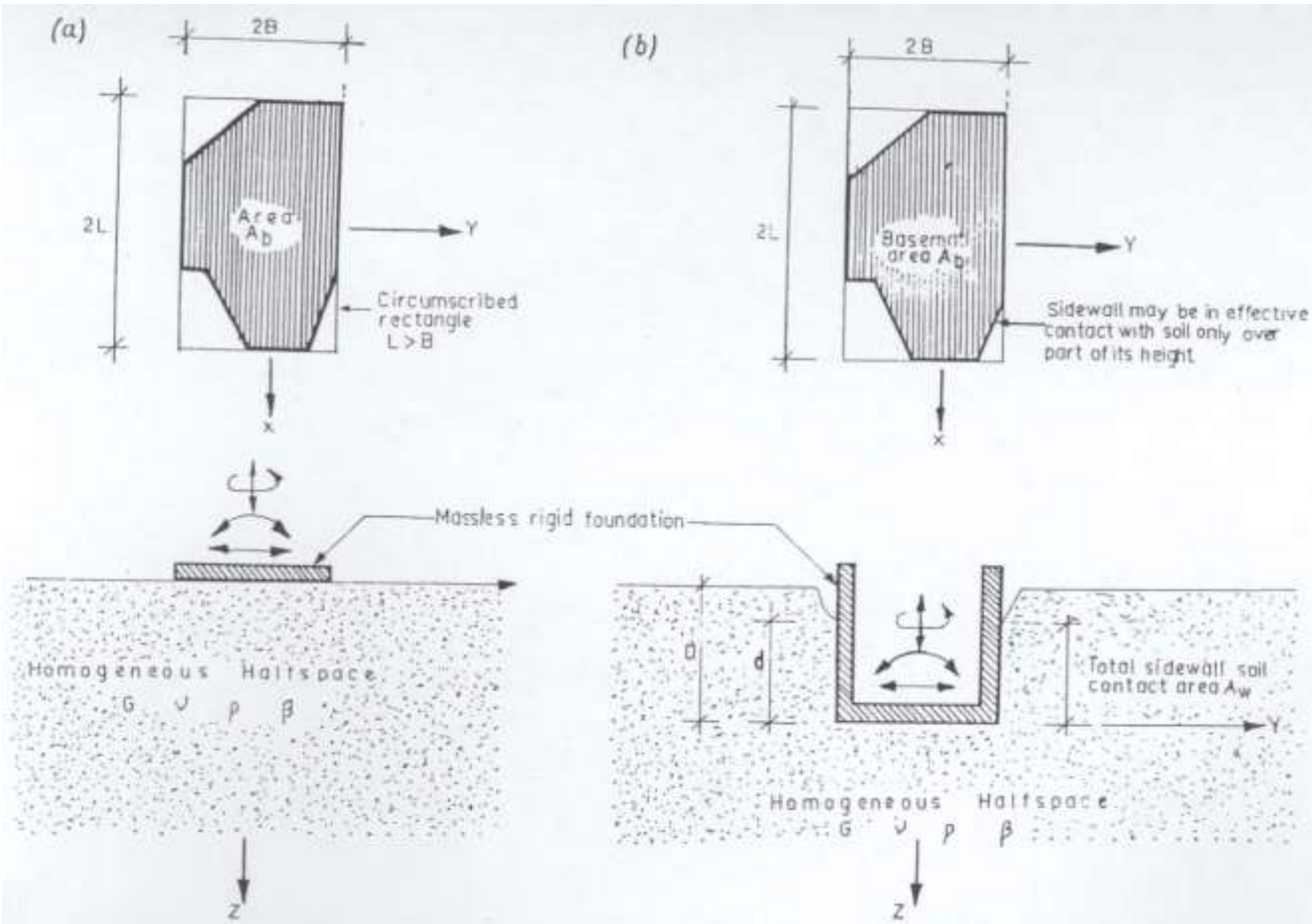


***Figure10** Foundations of arbitrary shape (a) surface foundation, (b) embedded foundation (Gazettas 1991b)*

Table 6. *Dynamic stiffness an damping coefficients for foundation of arbitrary shape resting on the surface of homogeneous half-space (Gazettas 1991b)*

Static stiffness, k (2)	**Dynamic stiffness coefficient, k (3)**	**Radiation dashpot coefficient, c (4)**
$K_z = [2GL/(1-\nu)](0.73+1.54x^{0.75})$ With $x = A_b/4L^2$	$k_z = k_z(L/B, \nu : a_0)$ is plotted in Fig. 11a	$C_z = (\rho V_{La} A_b) \bullet \bar{c}_z$ where $\bar{c}_z = \bar{c}_z(L/B_z a_0)$ is plotted in Fig. 11c
$K_y = [2GL/(2-\nu)](2+2.50x^{0.85})$	$k_y = k_y(L/B : a_0)$ is plotted in Fig. 11b	$C_y = (\rho V_s A_b)\bar{c}_y$ where $\bar{c}_y = \bar{c}_y(L/B; a_0)$ is plotted in Fig. 11d
$K_x = K_y - [0.2/(0.75-\nu)]GL[1-(B/L)]$	$k_x \cong 1$	$C_x = \rho V_s A_b$
$K_{rx} = [G/(1-e)]I_{bx}^{0.75}(L/B)^{0.25}[2.4+0.5(B/L)]$	$k_{rx} \cong 1 - 0.20a_0$	$C_{rx} = (\rho V_{La} I_{bx})\bar{c}_{rx}$ where $\bar{c}_{rx} = \bar{c}_{rx}(L/B; a_0)$ is plotted in Fig. 11e
$K_{ry} = [3G/(1-\nu)]I_{by}^{0.75}(L/B)^{0.15}$	$\nu < 0.40; k_{ry} \cong 1 - 0.26a_0$ $\nu \cong 0.50; k_{ry} \cong 1 - 0.26a_0(L/B)^{0.30}$	$C_{ry} = (\rho V_{La} I_{by})\bar{c}_{ry}$ where $\bar{c}_{ry} = \bar{c}_{ry}(L/B; a_0)$ is plotted in Fig. 11f
$K_t = 3.5GI_{bz}^{0.75}(B/L)^{0.4}(I_{bz}/B^4)^{0.2}$	$k_t = 1 - 0.14a_0$	$C_t = (\rho V_s I_{bz})\bar{c}_t$ where $\bar{c}_t = \bar{c}_t(L/B; a_0)$ is plotted in Fig. 11g

Equivalent spring for the surface footing for any mode of vibration can be obtained by multiplying he values of K in col. 2 with the corresponding values of k in col. 3.
Values of K in col. 2 and k in col.3 of this table are for calculating the equivalent soil springs by the impedance method only.
L, B and A_b are defined in Fig. 10. I_{bx},, I_{by} and I_{bz} represent the moment of inertia of the base area of the foundation about x, y and z-axis respectively.
$V_{La} = \frac{3.4}{\pi(1-\nu)} v_s$ is the apparent velocity of propagation of longitudinal waves.

Vibration mode (1)
Vertical (z)
Horizontal (y) (lateral direction)
Horizontal (x) (longitudinal direction)
Rocking (rx) about the longitudinal axis, x-axis
Rocking (rx) about the lateral, y-axis
Torsion (t)

Table 7. *Dynamic stiffness and damping coefficients of foundations of arbitrary shape embedded in half-space (Gazettas 1991b)*

Static stiffness, K_{emb} **(2)**	**Dynamic stiffness coefficient,** $K_{emb}(\omega)$ **(3)**	**Radiation dashpot coefficient,** $C_{emb}(\omega)$ **(4)**
$K_{z,emb} = K_z\left[1+(1/21)(D/B)(1+1.3x)\right]$ $\times\left[1+0.2(A_\omega/A_b)^{2/3}\right]$ Where $K_z \equiv K_{z,surface}$ is obtained from Table 4. A_ω =actual sidewall-soil contact area; for consultant effective contact height, d, along the perimeter: $A_\omega = (d)\times(perimeter);\ x = A_b/4L^2$	$(\nu \le 0.40)$: Fully embedded: $K_{z,emb} \cong K_z\left[1-0.09(D/B^{3/4}a_o^2\right]$ In a trench: $K_{z,tre} \cong K_z\left[1+0.09(D/B^{3/4}a_o^2\right]$ $(\nu \approx 0.48)$: Fully embedded with L/B = 1 – 2: $K_{z,emb} \cong K_z\left[1-0.09(D/B^{3/4}a_o^2\right]$ fully embedded with L/B > 3: $K_{z,tre} \cong K_z\left[1+0.09(D/B^{3/4}a_o^2\right]$ in a trench: $K_{z,tre} \cong K_z,\ where\ k_z = k_{z,surf}$ from table 6	$C_{z,emb} \cong C_z + \rho V_s A_\omega$ Where $C_z = C_{z,surface}$ is obtained form Table 4 and the associated chart of Fig. 11
$K_{y,emb} = K_y\left[1+0.15(D/B)^{0.5}\right]$ $\times\left\{1+0.52\left[(h/B)(A_\omega/L^2)\right]^{0.4}\right\}$ $K_{x,emb} = K_x.(K_{y,emb}/K_y)$ where $K_y \equiv K_{y,surface}$ and $K_x \equiv K_{x,surface}$ are obtained from Table 6	All v, partially embedded: interpolate $k_{y,emb}\ and\ k_{z,emb}$ can be estimated in terms of L/B, D/B and d/b for each a_o value of from the plots in Fig 12	$C_{y,emb} = C_y + 4\rho V_s Bd + 4\rho V_{La} Ld$ $C_{x,emb} = C_x + 4\rho V_{La} Bd + 4\rho V_{La} Ld$ where $C_y \equiv C_{y,surface}\ and\ C_x \equiv C_{x,surface}$ are obtained from Table 6 and the associated chart of Fig. 11
$K_{rx,emb} = K_{rx}\left[1+1.26(d/B)\right.$ $\left.\times\left[1+(d/B)(d/D)^{-0.2}(B/L)^{0.5}\right]\right]$ $K_{ry,emb} = K_{ry}\left[1+0.92(d/L)^{0.6}\right.$ $\left.\times\left[1.5+(d/L)^{1.9}(d/L)^{-0.6}\right]\right]$ where $K_{rx} \equiv K_{rx,surface}$ and $K_{ry} \equiv K_{ry,surface}$ are obtained from Table 6	$K_{rx,emb} \cong K_{rx}$ $K_{ry,emb} \cong K_{ry}$ The surface foundation k_{rx} and k_{ry} are obtained from Table 6	$C_{rx,emb} = C_{rx} + \rho I_{bx}(d/B)\left[V_{La}(d^2/B^2)+\right.$ $\left.3V_s + V_s(B/L)(1+(d^2/B^2))\right]\eta_r$ where $\eta_{rx} = 0.25+0.65\sqrt{a_0}\,(d/D)^{-a_0/2}(D/B)^{-1/4}$ $C_{ry,emb}$ is similarly evaluated from C_{ry} after replacing x by y and interchanging B with L in the foregoing two expressions. In both cases $a_0 = \omega B/V_s$

	Vibration mode (1)	Vertical (z)	Horizontal (y) and (x)	Rocking (rx) and (ry)

Table 7 *continued…*

Swaying-rocking (x-ry) x (y-rx) Torsion (r)	$K_{x-ry,emb} \cong (1/3)dK_{x,emb}$ $K_{y-rx,emb} \cong (1/3)dK_{y,emb}$ $K_{t,emb} = K_t . T_a . T_{tre}$ where is obtained from Table 6 $T_a = 1 + 0.4(D/d)^{0.5}(j_x/j_r)(B/D)^{0.6}$ $T_{tre} = 1 + 0.5(D/d)^{0.1}(B^4/I_{bz})^{0.13}$ $j_x = (4/3)d(B^3 + L^3) + 4BLd(L+B)$ $j_r = (4/3)BL(B^2 + L^2)$	$K_{x-ry,emb} \cong K_{y-rx,emb} \cong 1$ $k_{r,emb} \cong k_{t,surface}$	$C_{x-ry,emb} = (1/3)dC_{x,emb}$ $C_{y-rx,emb} = (1/3)dC_{y,emb}$ $C_{t,emb} = C_1 + 4pd\left[(1/3)V_{La}(L^3 + B^3) + V_s BL(L+B)\right]\eta_t$ where $C_t \equiv C_{t,surface}$ is obtained from Table 6 and Figure 11 $\eta_t \cong (d/D)^{-0.5} . a_0^2 / \left[a_0^2 + (l/2)(L/B)^{-1.5}\right]$

Equivalent soil spring for the embedded foundation for any mode of vibration is obtained by multiplying the values of k_{emb} in col.2 with the corresponding values of k_{emb} in col.3

The K_{emb} and k_{emb} given in cols. 2 and 3 respectively in this table are for calculating the equivalent soil springs by the impedance method only.

L, B, D, d, A_b and A_w are define in Figure 10

I_{bx}, I_{by} and I_{bz} represent the moment of inertia of the base area of the foundation about x, y and z axis respectively.

$V_{La} = \frac{3.4}{\pi(1-\nu)} v_s$ is the apparent velocity of propagation of longitudinal waves.

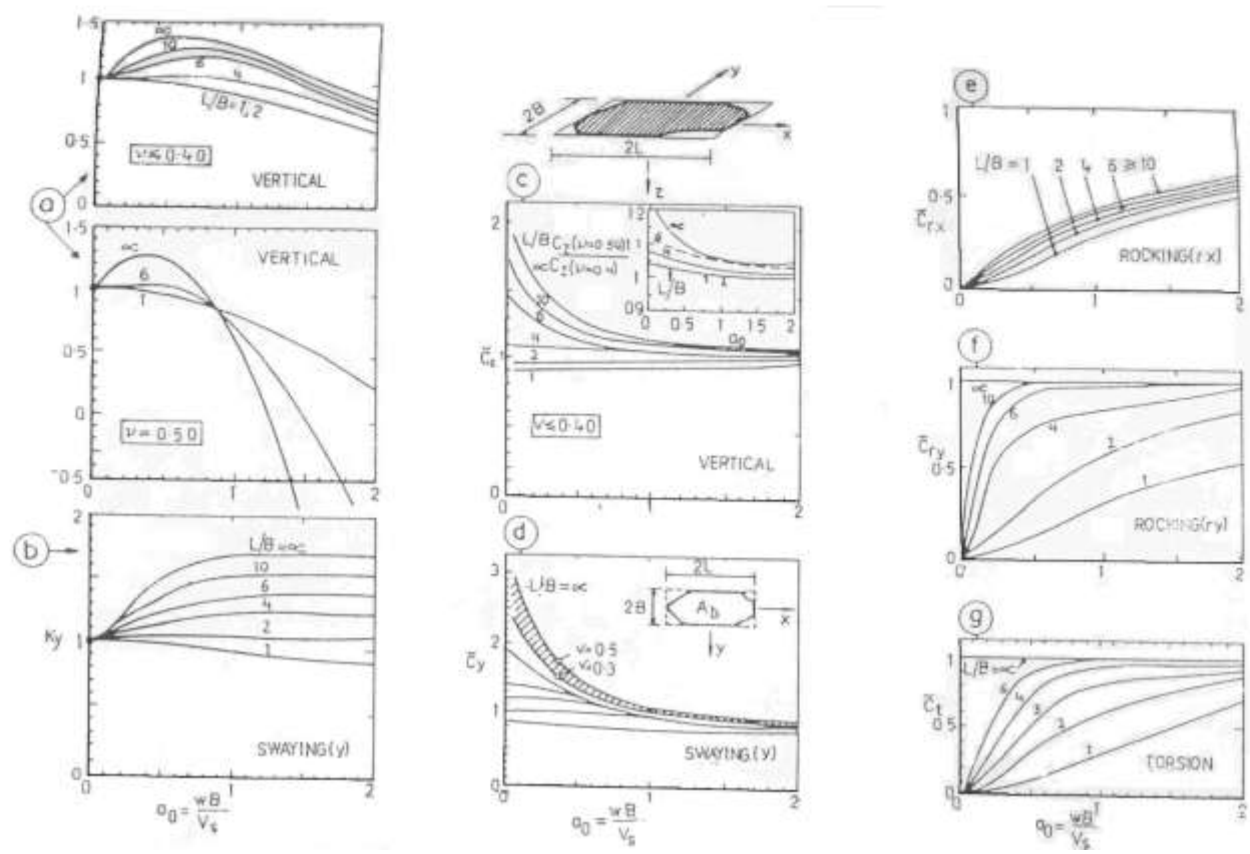


Figure 11. *Dimensionless graphs for determining dynamic stiffness and damping coefficients of surface foundations (accompanying* ***Table6****) (Gazettas, 1991b)*

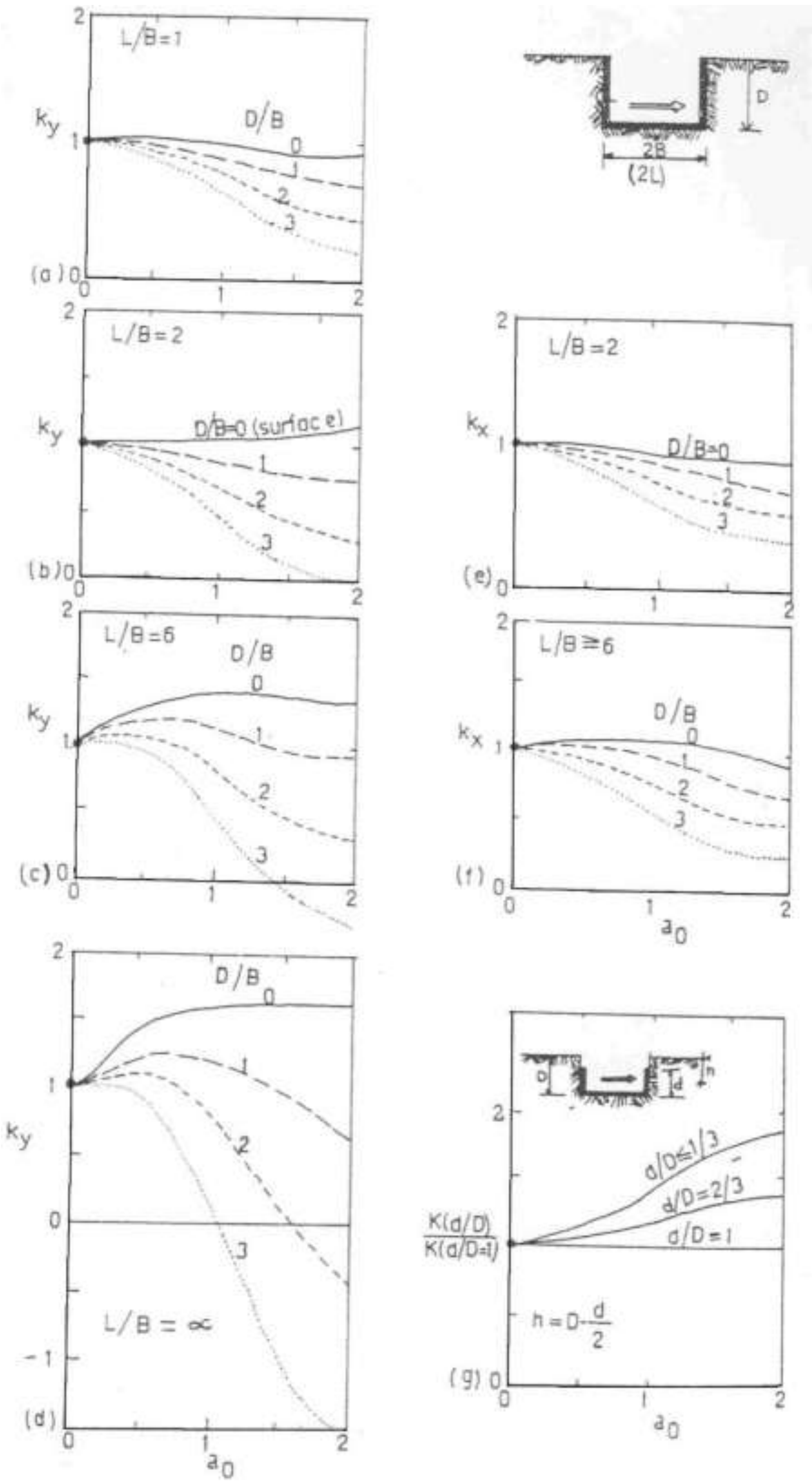


***Figure 12.** Dimensionless graphs or determining dynamic stiffness coefficients of embedded foundations (accompanying **Table 7**) (Gazettas, 1991b)*

COMPARISON OF PREDICTED AND OBSERVED RESPONSE

Very little information is available on comparison of measured response of machine foundations with theory. Such comparisons will increase the confidence of the designer. Richart and Whitman (1967) compared model footing test data with calculated values using the spring and damping obtained from the elastic half-space analog. The computed amplitudes of vertical vibrations were in the range of 0.5 to 1.5 times the observed values. Prakash and Puri (1981), however found that somewhat better agreement between computed and observed amplitudes is possible if the soil properties are selected after accounting for the effect of significant parameters such as mean effective confining pressure and strain amplitude.. Based on the results of the small-scale field experiments, Novak (1985) pointed out that the elastic half-space theory overestimates damping. Variation of soil properties and the presence of a hard stratum also influence the response of the footing. Adequate geotechnical investigations are necessary before meaningful comparisons of computed and predicted response can be made (Dobry and Gazettas 1985, Novak 1985).

Prakash and Puri (1981) compared the observed and computed response of a reciprocating compressor foundation which was undergoing excessive vibrations. The analysis of the compressor foundation was performed using the linear weightless spring method and also the elastic half-space analogs using soil properties for the as-designed condition and corresponding to the observed vibration amplitudes. The computed amplitudes by both the methods were far in excess of the permissible amplitudes as per manufacturer's specifications. The computed natural frequencies were found to be within about 25% of the observed natural frequencies in horizontal vibrations. Adequate soil exploration and a realistic determination of soil constants play an important role in the design of machine foundations.

Dobry et al, (1985) compared the observed response of model footing of different shapes with predictions made using the method proposed by Dobry and Gazettas (1985) for dynamic response of arbitrarily shaped foundations. They observed a strong influence of the footing shape on the stiffness and damping values. Gazettas and Stokoe (1991) compared results of 54 free vibration tests of model footing embedded to various depths in sand with theory. The model footing had rectangular, square and circular shapes. They observed that for the case of vertical vibrations and coupled rocking and sliding vibrations, the theory predicts reasonable values of damped natural frequencies provided the effective shear modulus is realistically chosen.

Manyando and Prakash (1991) reanalyzed the earlier data on circular footings (Fry 1963) considering nonlinearity of soil that is by using the values shear modulus corrected the effect mean effective confining pressure and shear strain induced in soil by the footing. Their analysis is essentially based on the concept of elastic-half space –analogs with modifications made for nonlinearity of soil.

The shear strain γ_z induced in the soil due to vertical vibrations was defined as below:

$$\gamma = \frac{A_{max}}{2B} \qquad 48$$

in which, A_{max} = amplitude of vertical vibrations and B= width of the foundation

Shear strain for torsional vibrations was considered to be equal to the rotational displacement at the edge of the base of the surface footing divided its radius. The shear strain for coupled rocking and sliding vibrations was considered as the rotation about the lateral axis of vibration through the combined center of gravity of the machine foundation system. The response of the surface footings was then predicted using equations 7,8, 14,15, 22,23,27 and 28 depending on the appropriate vibration mode and following an iterative procedure to account for the nonlinearity of soil. The effect of damping was also included in computations.

Typical results comparing the predicted and observed response of foundations for vertical, torsional and coupled rocking and sliding modes of vibrations are shown in Figs 13,14 and 15 respectively.

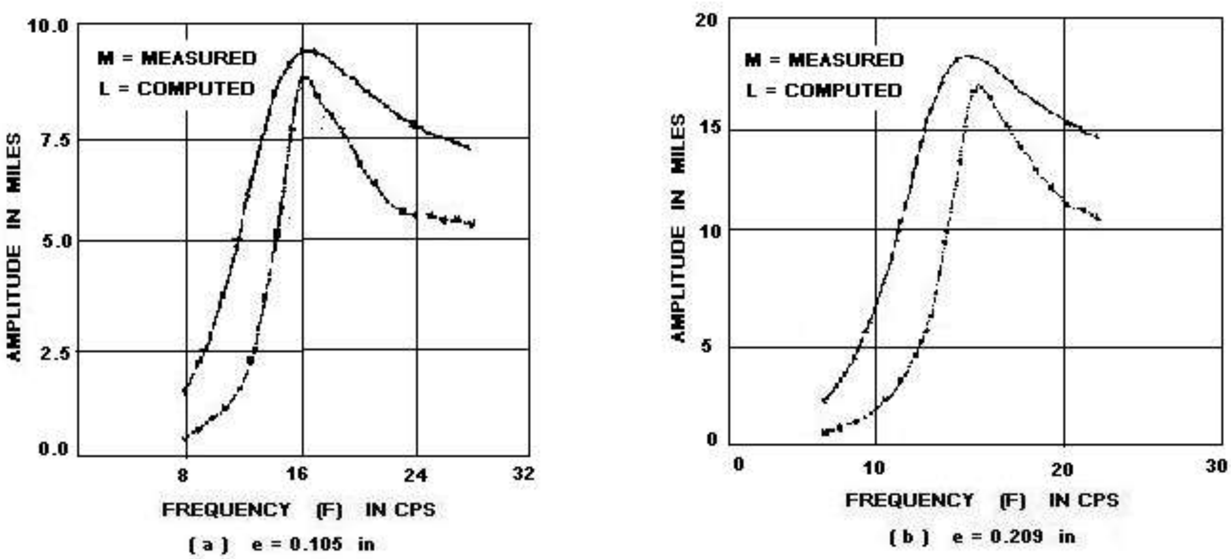
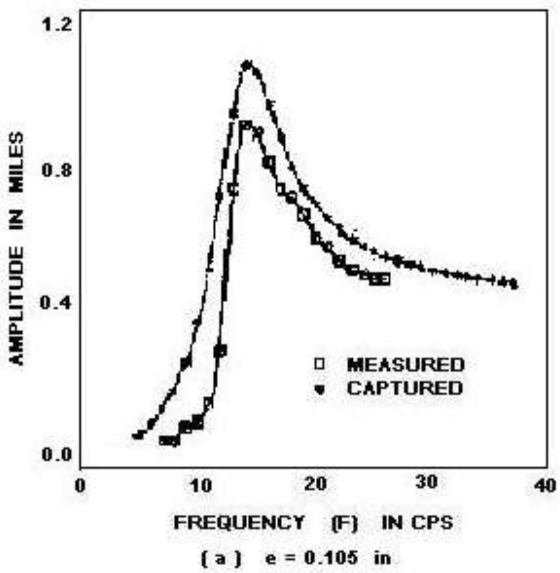


Figure 13. *Measured and predicted response of vertical vibration for different values of eccentricity (a) e = 0.105 and (b) e = 0.209 inches, Eglin, base 1-1 (Manyando & Prakash 1991)*



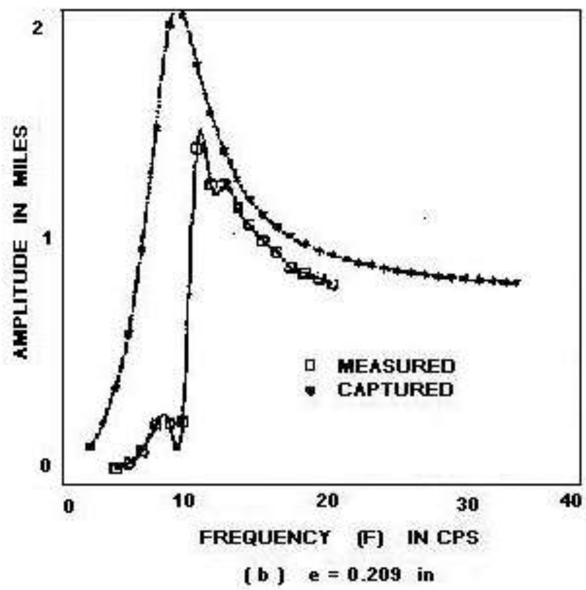
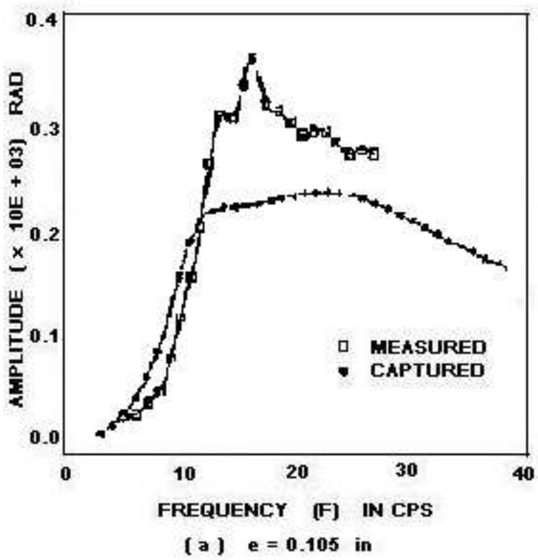


Figure 14. *Measured and predicted response of torsional vibrations for different values of eccentricity (a) e = 0.105 and (b) e = 0.209 inches, Eglin, base 1-1(Manyando & Prakash 1991)*



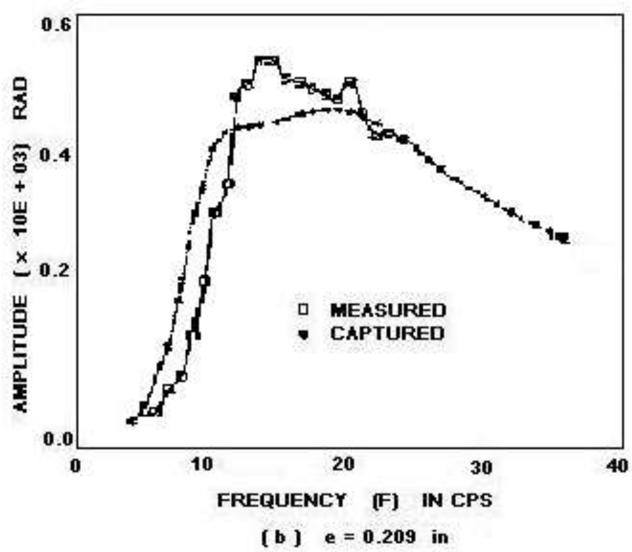


Figure 15. *Measured and predicted response of couples rocking and sliding vibrations for different values of eccentricity (a) e = 0.105 and (b) e = 0.209 inches, Eglin, base 1-1(Manyando& Prakash 1991)*

Figure 13 presents a comparison of the measured and computed response for the case of vertical vibrations . The general trend of the measured and computed response curves in Fig 13 (a,b) is similar. The predicted natural frequency of vertical vibration for the foundation under discussion shows good agreement with the measured natural frequency. Similar trend of data is observed for the case of torsional (Fig. 14, a and b) and for coupled rocking and sliding (Fig. 15, a and b). The computed amplitudes in all the cases are within about 20 to 50 % of the measured amplitudes.

Manyando and Prakash 1991) also investigated the role of geometrical and material damping on the comparison between measured and computed response. Based on their study it seems that natural frequencies are reasonably predicted by their model but more work is needed as for as prediction vibration amplitude is concerned. Prakash and Puri (1981) made a similar observation. Prakash and Tseng (1998) used frequency dependent stiffness and damping values to determine the response of vertically vibrating surface and embedded foundations. They compared the computed response with the reported data of Novak(1970). They observed that the radiation damping obtained from the elastic half space theory is generally over estimated and suggested factors for modification radiation damping..

SUMMARY

The methods for determination dynamic response of machine foundations subjected to harmonic excitation have been presented. Analogs based on the elastic half-space solutions are commonly used for their simplicity. The soil stiffness is generally considered frequency independent for design of machine foundations. Observations by several investigators have shown that the elastic half-space analog generally overestimates radiation damping. The impedance function method is a recent addition to the approaches available for design of machine foundations. The embedment of a foundation strongly influences its dynamic response.

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