

## DISCRETE WAVELET TRANSFORM USING CIRCULAR CONVOLUTION

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**Abstract**—Wavelet transform is widely used for filtering signals, for denoising any signal and for finding locations and distributions of singularities. In this paper there is a study of implementing discrete wavelet transform using circular convolution filter bank. Further we emphasize the advantage of using Perfect reconstruction filter bank structure. We further show how the PRCC analysis and synthesis filter banks are completely implemented in this domain and give a simple and a flexible method for implementation of these filters. We have represented the procedure of discrete wavelet transform with the help of matrices and tried to simplify complete process, after the theoretical description we are presenting a MATLAB code for implementing discrete wavelet transform via circular convolution matrix utilizing the filter bank structure utilizing the matrices.

**Keywords**- Discrete wavelet transform, filter bank structure, circular convolution, perfect reconstruction filter bank, conjugate mirror filters.

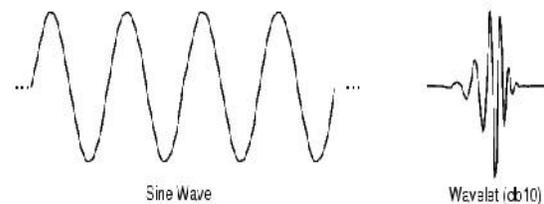
### I. INTRODUCTION

Although Fourier Transform is a dominant tool for frequency domain analysis. But it cannot provide information about spectrum with time and also signal is stationary, Moreover it can not locate drift trends, abrupt changes, beginning and end of events etc, and also the calculation are complex. Thus we move to wavelet transform. Thus in our paper we elaborate on wavelet transform, its type, further next section describes in detail the Discrete Wavelet Transform Filter bank structure their analysis and synthesis. The use of circular convolution in filter banks and their condition of being Perfect Reconstruction Filter bank are being described. After this the filter bank working downsampling and upsampling is being explained with the method of matrices, and in the

end MATLAB code for discrete wavelet transform via circular convolution is presented.

### II. WAVELET TRANSFORM

The wavelet transform is a tool that cuts up data, functions or operators into different frequency components, and then studies each component with a resolution matched to its scale.



It analysis uses little wavelike functions known as wavelet and it investigate the given signal into another form which is more useful form by scaling and shifting the given function. This phenomenon is called wavelet transform. Wavelet as a family of function constructed from a single function called "mother wavelet"  $\psi(t)$ . And expressed as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right), \quad a, b \in \mathbb{R}, a \neq 0 \dots\dots\dots(1)$$

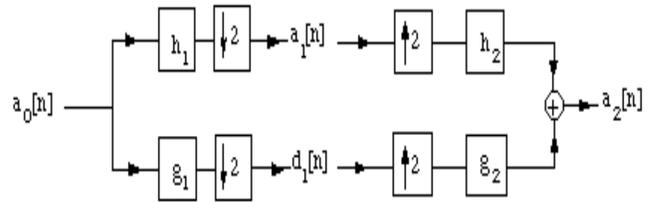
Where  $a$  is the scaling parameter and  $b$  is the shifting parameter.

#### A. Continuous Wavelet Transform (CWT)

The continuous wavelet transform provides a method for displaying and analyzing characteristics of signals that are dependent on time and scale. The CWT is an operator that takes a signal and produces a function of two variables: time and scale. As a two-variable function, it can be considered as a surface or image.

**B. Discrete Wavelet Transform (DWT)**

In particular, the DWT provides a sparse time-frequency representation of the original signal (the wavelet coefficients) that has the same number of samples as the original Signal .While the discrete Fourier transform is just the continuous Fourier transform estimated from a discrete grid of points, the difference between the CWT and DWT is more fundamental. The DWT is not constructed by estimating the CWT at discrete points.It is generally the basis functions of the DWT basis functions are complete and orthogonal. The DWT allows for an orthogonal projection on a complete set of modes and thus, the wavelet coefficient is independent .The discrete modal is best approached through the mechanism of filter bank. A filter bank give rise to a discrete wavelet transform (DWT) similar to the discrete fourier transform.The “discrete wavelets” appear as pure waveforms associated the DWT just as the exponential pure wave form arose from the rows of the fourier transform matrix. DWT is actually a “Subset” of scale and position based on power of two ,rather than every “possible” set of scale and position in continuous wavelet transform. It.behaves like a filter bank: signal in, coefficients out where Down-sampling is necessary.



The filter bank is said to be a *perfect reconstruction filter bank* when  $a_2 = a_0$  . If, additionally,  $h = h_2$  and  $g = g_2$ , the filters are called *conjugate mirror filters*.

Perfect reconstruction filter banks are characterized in a theorem by Vetterli. When the filters have a finite impulse response, the  $g$  and  $g_2$  filters can easily be derived from the  $h$  and  $h_2$  filters, and the filter synthesis is equivalent to solving

$$\hat{h}_2^*(\omega) \hat{h}_2(\omega) + \hat{h}_2^*(\omega + \pi) \hat{h}_2(\omega + \pi) = 2.$$

where  $h$  and  $h_2$  are trigonometric polynomial.

The transfer function of filter bank is thus given as

$$\bar{T}(z) = \frac{1}{2} [H_0(z)G_0(z) + H_1(z)G_1(z)] = \frac{1}{4} (1+z^{-1})(1+z^{-1}) + \frac{1}{4} (1-z^{-1})(-1+z^{-1}) = z^{-1}$$

Therefore this FB is a perfect reconstruction system

**III. FILTER BANK**

**Definition**

A (discrete) two-channel multirate filter bank convolves a signal  $a_0$  with a low-pass filter  $h_1[n] = h[-n]$  and a high-pass filter  $g_1[n] = g[-n]$  and then subsamples the output:

$$a_1 [n] = a_0 * h_1 [2n] \text{-----}(2)$$

and

$$d_1 [n] = a_0 * g_1 [2n] \text{-----}(3)$$

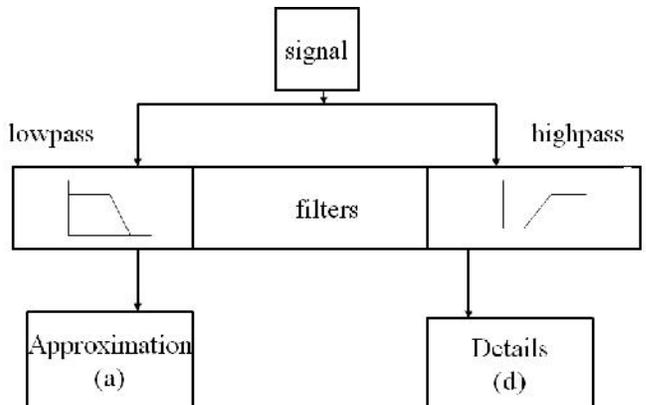
A reconstructed signal  $a_2$  is obtained by filtering the zero expanded signals with a dual low-pass filter  $h_2$  and a dual high-pass filter  $g_2$ . If  $z(x)$  denotes the signal obtained from  $x$  by inserting a zero between every sample, this can be written as:

$$a_2 [n] = z(a_1) * h_2 [n] + z(d_1) * g_2 [n] \text{-----}(4)$$

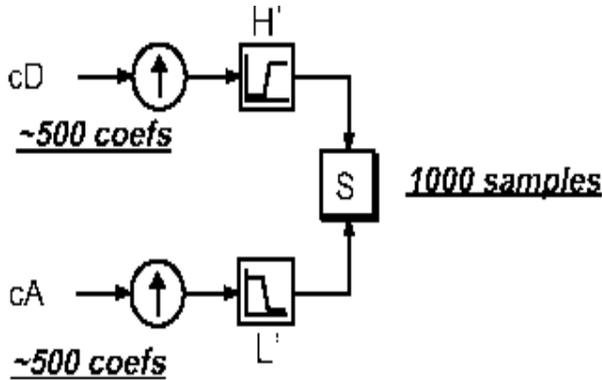
The following figure illustrates the decomposition and reconstruction process.

**IV. THE FILTER BANK, ANALYSIS AND SYNTHESIS**

Approach adopted by filter bank method is to split the signal into various frequency bands. The first step is to separate the high and low frequency component. Now given a pair of low and high pass filters,  $l$  and  $h$  we can define a decomposition of a signal into a low and highpass components.



Wavelet synthesis similarly combine the coefficients of approximation and detail filtered output to give double the samples, as shown CD and CA coefficients combine to give their double samples



Let us illustrate this with a specific example. For our low pass and high pass filter we take the familiar two point averager and two point differencer.

$$l(0) = 1/2, l(1) = 1/2, l(r) = 0, \text{ otherwise}$$

$$h(0) = 1/2, h(1) = 1/2, h(r) = 0, \text{ otherwise}$$

for a vector X we get the low pass  $l * x$  and high pass  $h * X$  filtered data. To be explicit for our choice of  $l$  and  $h$  we get:

$$M_l X(r) = l * X(r) = (X(r) + X(r-1)) / 2 \text{ mod } N$$

$$M_h X(r) = h * X(r) = (X(r) - X(r-1)) / 2 \text{ mod } N$$

Where  $M_l$  and  $M_h$  are the appropriate convolution matrices. We call  $l * X$  the smooth approximation to X and  $h * X$  the high resolution detail of X.

To analyze the filter bank concept further, let's pick  $N = 8$  and walk through all the details of this example

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix},$$

From this we get

$$l * X = M_l X = \frac{1}{2} \begin{bmatrix} x_1 + x_7 \\ x_2 + x_1 \\ x_3 + x_2 \\ x_4 + x_3 \\ x_5 + x_4 \\ x_6 + x_5 \\ x_7 + x_6 \end{bmatrix}, \quad h * X = M_h X = \frac{1}{2} \begin{bmatrix} x_0 - x_7 \\ x_1 - x_0 \\ x_2 - x_1 \\ x_3 - x_2 \\ x_4 - x_3 \\ x_5 - x_4 \\ x_6 - x_5 \\ x_7 - x_6 \end{bmatrix}$$

From the vectors it is easy to see that we can reconstruct X from  $l * X$  and  $h * X$ :

$$X = l * X + h * X$$

The only problem with the above process is that we have created twice as much data as we started with. However when N is even, we can eliminate half of the filtered data and still be able to recover the original data, if we throw away every second

From component of  $M_l X$  and  $M_h X$  we get two vectors

$$C_l = D M_l X = \frac{1}{2} \begin{bmatrix} x_0 + x_7 \\ x_2 + x_1 \\ x_4 + x_3 \\ x_6 + x_5 \end{bmatrix},$$

$$C_h = D M_h X = \frac{1}{2} \begin{bmatrix} x_0 - x_7 \\ x_2 - x_1 \\ x_4 - x_3 \\ x_6 - x_5 \end{bmatrix}.$$

Where D is the downsampling operator.

A vector C can be generated

$$C = \begin{bmatrix} C_\ell \\ C_h \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x_0 + x_7 \\ x_2 + x_1 \\ x_4 + x_3 \\ x_6 + x_5 \\ x_0 - x_7 \\ x_2 - x_1 \\ x_4 - x_3 \\ x_6 - x_5 \end{bmatrix} .$$

We recover addition and subtraction of down-sampled data the original data though slightly scrambled.

$$C_\ell + C_h = \frac{1}{2} \begin{bmatrix} x_0 + x_7 \\ x_1 + x_2 \\ x_3 + x_4 \\ x_5 + x_6 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x_0 - x_7 \\ -x_1 + x_2 \\ -x_3 + x_4 \\ -x_5 + x_6 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_2 \\ x_4 \\ x_6 \end{bmatrix} ,$$

$$C_\ell - C_h = \frac{1}{2} \begin{bmatrix} x_0 + x_7 \\ x_1 + x_2 \\ x_3 + x_4 \\ x_5 + x_6 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} x_0 - x_7 \\ -x_1 + x_2 \\ -x_3 + x_4 \\ -x_5 + x_6 \end{bmatrix} = \begin{bmatrix} x_7 \\ x_1 \\ x_3 \\ x_5 \end{bmatrix} .$$

Such objects can be implemented in hardware in a variety of ways .While the scrambled data can be fixed by companion synthesis filter.

### V. MATLAB IMPLEMENTATION OF DWT BY CIRCULAR CONVOLUTION

The wavelet tool box has two different implementations of the wavelet transform. The definition depends on which signal model we are using , and whether we are using infinite convolution or cyclic,since we are dealing with cyclic ie,circular convolution The one corresponding to our theory is DWT by circular convolution also termed as periodic case. In MATLAB version the DWT is always defined even if the matrices have odd size lengths. The down sampling and up sampling is done slightly differently.

### VI. THE MATLAB CODE FOR DISCRETE WAVELET TRANSFORM VIA CIRCULAR

#### Convolution Matrix

```
>> disp('== GENERALIZED MATRICIAL DWT ==')
>> %% STEP 1 - Base Filters
>> typeofbasis = 'o';
>> typbior = 'bior2.2';
>> typor = 'db2';
>> % Obtain base filters
>> if(typeofbasis == 'b')
[h0,h1,g0,g1] = biorfilt(Df,Rf);
```

```
[Rf,Df] = biorwavf(typbior);
[h0,h1,g0,g1] = biorfilt(Df,Rf);
elseif (typeofbasis == 'o')
[h0,h1,g0,g1] = wfilters(typor);
end;
```

```
>> %% STEP 2 - Parameter configuration
>> N = 16; %Number of input samples
>> x = (1:N)';
>> L = length(h0); %Base filter lenght
>> n_stages = 2; %Of the low-pass stage
>> n_branches = 2^n_stages; %Number of branches
>> dec_fact = n_branches; %Decimating factor
>> fprintf('\nCircular convolution processing\n');
```

#### Circular convolution processing

```
>> hx = formfilter(n_stages, 1, h0, h1);

>> Lhx = length(hx);

>> if Lhx>N
beep; beep; beep;
disp('ERROR: Input vector must be longer, increase n');
disp('End of program ### ');
return;
end

>> movil_1 = [fliplr(h1),zeros(1,N-L),fliplr(h1),zeros(1,N-L)];
>> lm = length(movil_1);
>> dec_fact = 2;
>> rowsperfilter = N/dec_fact; % Square matrix gets decimated
>> W1 = zeros(rowsperfilter,N); %MALLOC
```

```

>> for i = 0:(N/2)-1
i;
W1(i+1,:) = movil_1(lm-(N)-(i*dec_fact)+1:lm-
(i*dec_fact)); %Rama pasaaltas
end
>> disp('High-pass matrix: ');
High-pass matrix:
>> W1

W1 =

>> y1 = W1*x;
>> % Low-Pass n-branch Analysis Circular
convolution matrix
>> dec_fact = 2^n_stages;
>> rowsperfilter = N/dec_fact
>> % Each branch is assigned to a page of that array
>> W0 = zeros(rowsperfilter, N, n_branches);
%MALLOC
>> Y0 = zeros(rowsperfilter, n_branches);
%MALLOC
>> % Matrix generation
>> for i = 0:(n_branches/2)-1
if n_stages < 3
hx = formfilter(n_stages, (n_branches/2) - i, h0, h1);
else
hx = formfilter(n_stages, (n_branches/2) - i - 1, h0,
h1);
end
Lhx = length(hx);
movil = [fliplr(hx),zeros(1,N-
Lhx),fliplr(hx),zeros(1,N-Lhx)];
lm = length(movil);
for j = 0:rowsperfilter-1
W0(j+1, :, i+1) = movil(lm-(dec_fact*j)-N + 1 : lm-
(dec_fact*j));
end
Y0(:,i+1) = W0(:,i+1)*x;
End

>> %% Low-pass part recovering code
>> xnm = zeros(N, n_branches/2); %MALLOC
>> for i=0:(n_branches/2)-1
xnm(:,i+1) = ((W0(:,i+1))) * Y0(:,i+1);
end
>> [n,m] = size(xnm);
>> xn1 = (W1')*y1; %High-pass part recovering
>> xn = zeros(n,1);
>> % Sum between the low-pass and high-pass
recovered parts

```

```

>> for i=1:n
xn(i,1) = sum(xnm(i,:)) + xn1(i);
end
>> disp('Original signal');
Original signal
>> disp(x)
>> disp('Recovered signal');
Recovered signal
>> disp(xn)
>> err = (x - xn).^2;
>> disp('Error :');
Error
>> err = (sum(err)/N);
>> disp(err)

```

## VII. CONCLUSION

In this paper, we explained the idea of filter bank structure being utilized for generating Discrete wavelet transform, the convolution involved is circular convolution for finite length signal. Here we used the MATLAB program for implementing our aim using low pass and high pass filters generated through filter bank structure, downsampling and upsampling utilizing circular convolution. Through this method Discrete Wavelet Transform via circular convolution is performed successfully with a small % of error, which is obvious due to practical constraints. The error is being tried to be minimized by using PRCC filter banks.

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