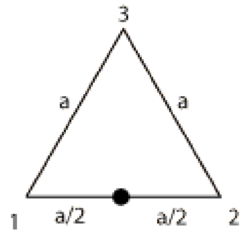
Homework 3
MAE 118C

Problems 2, 5, 7, 10, 14, 15, 18, 23, 30, 31 from Chapter 5, Lamarsh & Baratta

2.



Point sources emit S nuetrons/sec at points, 1, 2, and 3 find the flux current half way between one side of the triangle (black dot).

The flux for a point source is: $\phi = \dfrac{S}{4\pi r^2}$ from problem 1.

The flux is then the sum of the point sources fluxes.

$$\phi = \frac{2 \cdot S}{4\pi\left(\frac{a}{2}\right)^2} + \frac{S}{4\pi\left[a^2 - \left(\frac{a}{2}\right)^2\right]}$$

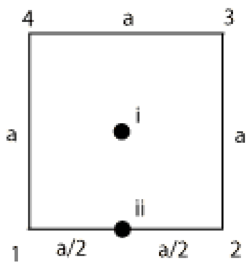
The first term is the flux from points 1 and 2 which are equal the second term is the flux from point 3.

The current from points 1 and 2 cancle so you only need to calculate the current from point 3.

$$J = -D \cdot \left(\frac{d}{dr}\phi\right) = \frac{DS}{2\pi\left[a^2 - \left(\frac{a}{2}\right)^2\right]^{\frac{3}{2}}}$$

The current has a direction which is in the direction of the vector from point 3 to the point you are calculating the flux.

5.



This case in an infinite moderator so using the flux equation 5.33.

At the center of the square the current is zero due to symetry. The flux is simply the sum of the 4 point sources.

$$\phi_i = \frac{\sqrt{2} \cdot S e^{\frac{-\sqrt{2}\,a}{2L}}}{\pi D \cdot a}$$

The flux at point ii is again the sum of the fluxes.

$$\phi_{ii} = \frac{S e^{\frac{-a}{2L}}}{\pi D \cdot a} + \frac{S e^{\frac{-\sqrt{a^2 - \left(\frac{a}{2}\right)^2}}{L}}}{2\pi D \cdot \sqrt{a^2 - \left(\frac{a}{2}\right)^2}}$$

The current from each point is just,

$$J = -D \cdot \left(\frac{d}{dr}\phi\right) = \frac{S}{4\pi} \cdot \left(\frac{1}{rL} + \frac{1}{r^2}\right) e^{\frac{-r}{L}}$$ which is given 2 equations before 5.33

The currents from point 1 and 2 cancel.

Looking at the diagram, the horizonal portion of the currents from poitns 3 and 4 cancel. The sum of the vertical currents add with their direction being down in the diagram.

Distance from point 3 to ii = $\sqrt{a^2 + \left(\frac{a}{2}\right)^2}$ same for 4 to ii.

$$J_3 = \frac{S}{4\pi} \cdot \left[\frac{1}{L \sqrt{a^2 + \left(\frac{a}{2}\right)^2}} + \frac{1}{\left[a^2 + \left(\frac{a}{2}\right)^2\right]}\right] e^{\frac{-\sqrt{a^2 + \left(\frac{a}{2}\right)^2}}{L}} = J_4$$ current from point 3 and 4 at ii

$$J = \frac{2S}{4\pi} \cdot \left[\frac{1}{L \sqrt{a^2 + \left(\frac{a}{2}\right)^2}} + \frac{1}{\left[a^2 + \left(\frac{a}{2}\right)^2\right]}\right] e^{\frac{-\sqrt{a^2 + \left(\frac{a}{2}\right)^2}}{L}} \cdot \sin\left[\tan^{-1}(2)\right]$$ in the downward direction

Where the sine and tangent terms come from forming a right triangle with points ii, 2, 3 and then using the ratio of the sides to find the angle at point ii and then using that angle to find the portion of the current in the downward direction.

7.

Point source in an infinite moderator,

$$\phi = \frac{S e^{\frac{-r}{L}}}{4\pi D \cdot r}$$

$$J = -D \cdot \left(\frac{d}{dr}\phi\right) = \frac{S}{4\pi} \cdot \left(\frac{1}{rL} + \frac{1}{r^2}\right) e^{\frac{-r}{L}}$$

a) To find the number of neutrons passing through a surface, find the current at the surface then integrate over the surface area.

$$J = \frac{S}{4\pi} \cdot \left(\frac{1}{rL} + \frac{1}{r^2}\right) e^{\frac{-r}{L}}$$ where r is the radius of the sphere which has area $4\pi r^2$

total number of neutrons = $r^2 \cdot S \cdot \left(\frac{1}{rL} + \frac{1}{r^2}\right) e^{\frac{-r}{L}}$

b) The number absorbed per second within the sphere is equal to

$\int \Sigma_a \cdot \phi \, dV$ in our case the volume integral is over the volume of the sphere

$$\int \Sigma_a \cdot \phi \, dV = 4\pi \int_0^r \frac{r^2 \cdot S \cdot e^{\frac{-r}{L}}}{4\pi D \cdot r} dr = -\frac{L \cdot \Sigma_a \cdot S \cdot \left(r \cdot e^{-\frac{r}{L}} - L + L \cdot e^{-\frac{r}{L}}\right)}{D}$$

c) Verify the continuity equation, the number aborbed + the number leaked equals the number produced.

$$D = L^2 \cdot \Sigma_a$$

$$r^2 \cdot S \cdot \left(\frac{1}{rL} + \frac{1}{r^2}\right) e^{\frac{-r}{L}} - \frac{L \cdot \Sigma_a \cdot S \cdot \left(r \cdot e^{-\frac{r}{L}} - L + L \cdot e^{-\frac{r}{L}}\right)}{L^2 \cdot \Sigma_a} = S$$

10.

Current equals zero by symetry, and the flux can have no spacial dependence due to symetry.

Diffusion equation = $$\nabla^2\phi - \frac{1}{L^2}\phi = \frac{-S}{D}$$ Eq (5.19)

flux must be constant $\phi = C$

plug into diffusion equation, $$\frac{-C}{L^2} = \frac{-S}{D} \qquad C = \frac{S \cdot L^2}{D} = \frac{S}{\Sigma_a}$$

$$\phi = \frac{S}{\Sigma_a}$$

14.

Again using the diffusion equation, $\nabla^2\phi - \frac{1}{L^2}\phi = \frac{-S}{D}$

There is now a non-uniformity in r so the solutions can vary in the r direction. A general solution to the diffusion equation is,

$$\phi = A \cdot \frac{\sinh\left(\frac{-r}{L}\right)}{r} + B \cdot \frac{\cosh\left(\frac{r}{L}\right)}{r} + C$$

Now using boundry conditions, the flux needs to be finite at r = 0 so B = 0. Note that the first term does NOT go to infinity as r goes to zero, this can be seen by expanding sinh as an infinite series.

Again we have, $$\frac{-C}{L^2} = \frac{-S}{D} \qquad C = \frac{S \cdot L^2}{D} = \frac{S}{\Sigma_a}$$

$$\phi = A \cdot \frac{\sinh\left(\frac{-r}{L}\right)}{r} + \frac{S}{\Sigma_a}$$

Now using, $\phi(R + d) = 0$ $$0 = A \cdot \frac{\sinh\left(\frac{-R - d}{L}\right)}{(R + d)} + \frac{S}{\Sigma_a}$$

$$A = \frac{-S}{\Sigma_a} \cdot \frac{(R + d)}{\sinh\left(\frac{-R - d}{L}\right)}$$

$$\phi = \frac{-S}{\Sigma_a} \cdot \frac{(R + d)}{\sinh\left(\frac{-R - d}{L}\right)} \cdot \frac{\sinh\left(\frac{-r}{L}\right)}{r} + \frac{S}{\Sigma_a} = \frac{S}{\Sigma_a}\left[1 - \frac{(R + d)}{r} \cdot \frac{\sinh\left(\frac{-r}{L}\right)}{\sinh\left(\frac{-R - d}{L}\right)}\right]$$

using, $\sinh(-x) = -\sinh(x)$

$$\phi = \frac{S}{\Sigma_a}\left[1 - \frac{(R + d)}{r}\cdot\frac{\sinh\left(\frac{r}{L}\right)}{\sinh\left(\frac{R + d}{L}\right)}\right]$$

b)

$$J = -D\cdot\left(\frac{d}{dr}\phi\right) = -D\cdot\left[\frac{d}{dr}\left[\frac{S}{\Sigma_a}\left[1 - \frac{(R + d)}{r}\cdot\frac{\sinh\left(\frac{r}{L}\right)}{\sinh\left(\frac{R + d}{L}\right)}\right]\right]\right]$$

$$J = -D\cdot\frac{S}{L}\cdot\frac{(R + d)}{\sinh\left(\frac{R + d}{L}\right)}\cdot\left(\frac{\cosh\left(\frac{r}{L}\right)}{L\cdot r} - \frac{\sinh\left(\frac{r}{L}\right)}{r^2}\right)$$

c) How many neutrons leak from the sphere?

$$J(R)\cdot 4\pi R^2$$

$$\frac{D\cdot 4\pi R^2 S}{L}\cdot\frac{(R + d)}{\sinh\left(\frac{R + d}{L}\right)}\cdot\left(\frac{\sinh\left(\frac{R}{L}\right)}{R^2} - \frac{\cosh\left(\frac{R}{L}\right)}{L\cdot R}\right)$$

d)

Average probability that a source neutron will leave the sphere is the number that leak divided by the total number produced

$$\frac{\frac{D\cdot 4\pi R^2 S}{L}\cdot\frac{(R + d)}{\sinh\left(\frac{R + d}{L}\right)}\cdot\left(\frac{\sinh\left(\frac{R}{L}\right)}{R^2} - \frac{\cosh\left(\frac{R}{L}\right)}{L\cdot R}\right)}{S}$$

$$\frac{D\cdot 4\pi R^2}{L}\cdot\frac{(R + d)}{\sinh\left(\frac{R + d}{L}\right)}\cdot\left(\frac{\sinh\left(\frac{R}{L}\right)}{R^2} - \frac{\cosh\left(\frac{R}{L}\right)}{L\cdot R}\right)$$

15.

$$J = -D\cdot\left(\frac{d}{dr}\phi\right)$$

$$\phi_i = A_i\cdot\frac{\sin\left(\frac{\pi\cdot r}{R}\right)}{r} \qquad J = -D\cdot A_i\cdot\left(\frac{\pi\cdot\cos\left(\frac{\pi\cdot r}{R}\right)}{R\cdot r} - \frac{\sin\left(\frac{\pi\cdot r}{R}\right)}{r^2}\right)$$

$$R_1 := 50cm$$

Leakage = L

$$L_1 := -2.2cm\cdot 3\cdot 10^{15}\frac{1}{cm\cdot s}\cdot\left(\frac{\pi\cdot\cos(\pi)}{R_1{}^2} - \frac{\sin(\pi)}{R_1{}^2}\right)\cdot 4\pi\cdot R_1{}^2 = 2.606\times 10^{17}\frac{1}{s}$$ neutrons per second

$$L_2 := -1.7cm\cdot 2\cdot 10^{16}\frac{1}{cm\cdot s}\cdot\left(\frac{\pi\cdot\cos(\pi)}{R_1{}^2} - \frac{\sin(\pi)}{R_1{}^2}\right)\cdot 4\pi\cdot R_1{}^2 = 1.342\times 10^{18}\frac{1}{s}$$ neutrons per second

$$L_3 := -1.05cm\cdot 1\cdot 10^{16}\frac{1}{cm\cdot s}\cdot\left(\frac{\pi\cdot\cos(\pi)}{R_1{}^2} - \frac{\sin(\pi)}{R_1{}^2}\right)\cdot 4\pi\cdot R_1{}^2 = 4.145\times 10^{17}\frac{1}{s}$$ neutrons per second

18.

$$J = -D\cdot\nabla\phi$$

$$\phi_T = A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{a}\right)\cos\left(\frac{\pi z}{a}\right)$$

Can't figure out how to make it a tilda so just calling it a.

$$J = -D\left(\frac{d}{dx}\phi_T\cdot\vec{x} + \frac{d}{dy}\phi_T\cdot\vec{y} + \frac{d}{dz}\phi_T\cdot\vec{z}\right)$$

where $\vec{x}$ is the unit vector in the x direction

a) $$J = -DA\frac{\pi}{a}\cdot\cos\left(\frac{\pi x}{a}\right)\cdot\cos\left(\frac{\pi y}{a}\right)\cdot\cos\left(\frac{\pi z}{a}\right)\cdot\left(\frac{\sin\left(\frac{\pi\cdot x}{a}\right)}{\cos\left(\frac{\pi x}{a}\right)}\cdot\vec{x} + \frac{\sin\left(\frac{\pi\cdot y}{a}\right)}{\cos\left(\frac{\pi y}{a}\right)}\cdot\vec{y} + \frac{\sin\left(\frac{\pi\cdot z}{a}\right)}{\cos\left(\frac{\pi z}{a}\right)}\cdot\vec{z}\right)$$

b)

Evaluate J dotted into x hat at x = a/2 and then integrate from y = -a/2 to a/2 and z = -a/2 to a/2.

$$J\left(\frac{a}{2},y,z\right) = -DA\frac{\pi}{a}\cdot\cos\left(\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi y}{a}\right)\cdot\cos\left(\frac{\pi z}{a}\right)\cdot\left(\frac{\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)}\cdot\vec{x} + \frac{\sin\left(\frac{\pi\cdot y}{a}\right)}{\cos\left(\frac{\pi y}{a}\right)}\cdot\vec{y} + \frac{\sin\left(\frac{\pi\cdot z}{a}\right)}{\cos\left(\frac{\pi z}{a}\right)}\cdot\vec{z}\right)$$

$$J\left(\frac{a}{2}, y, z\right)\cdot\vec{x} = -DA\frac{\pi}{a}\cos\left(\frac{\pi y}{a}\right)\cdot\cos\left(\frac{\pi z}{a}\right)\vec{x}$$

$$\int_{\frac{-a}{2}}^{\frac{a}{2}} \cos\left(\frac{\pi y}{a}\right) dy \rightarrow \frac{2\cdot a}{\pi}$$

Leaking neutrons = $-DA\frac{\pi}{a}\cdot\frac{2\cdot a}{\pi}\cdot\frac{2\cdot a}{\pi} = \frac{-D\cdot A\cdot 4a}{\pi}$

per side

c)

By symetry, the total number is just the answer from part b) times 6.

$$\frac{-D\cdot A\cdot 24\cdot a}{\pi}$$

23.

a) $b := 940\cdot 10^{-24}\mathrm{cm}^2$

$$N_A := .6022\cdot 10^{24}\cdot\frac{1}{\mathrm{mol}}$$

$$b\cdot\frac{1}{\mathrm{cm}^2\cdot\mathrm{s}}\cdot\frac{N_A}{6.015\,\frac{\mathrm{gm}}{\mathrm{mol}}} = 2.97\times 10^9\cdot\frac{1}{\mathrm{gm}\cdot\mathrm{yr}}$$

b)

$2.28\cdot 10^9\cdot\frac{1}{\mathrm{yr}}\cdot 1\cdot 10^{14} = 7.225\times 10^{15}\frac{1}{\mathrm{s}}$ decays per second

$\frac{7.225\cdot 10^{15}}{3.7\cdot 10^{10}} = 1.953\times 10^5$ curies

30.

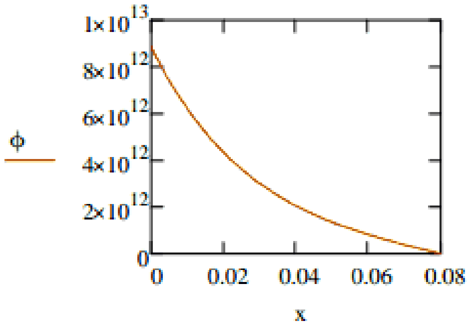
$N_1 := 200$ $\quad i := 0..N_1$ $\quad n := 0..N_1$

$$x_i := \frac{8\cdot i\cdot\mathrm{cm}}{N_1}$$

$L_T := 2.85\mathrm{cm}$ $\quad D := .16\mathrm{cm}$ $\quad a := 8\mathrm{cm}$ $\quad S_1 := 10^8\cdot\frac{1}{\mathrm{cm}^2\cdot\mathrm{s}}$

$\Sigma_a := .0197\cdot\frac{1}{\mathrm{cm}}$

$$\phi := \frac{S_1 \cdot L_T}{2 \cdot D} \cdot \left[\frac{\sinh\left[\frac{(a - x)}{L_T}\right]}{\cosh\left(\frac{a}{L_T}\right)} \right]$$



where x is in units of meters and
the flux is in units of neutrons/cm^2*s.

31.

Using Eq. 5.64 to calculate the change in the thermal diffusion parameter

$$L_T^2(\rho, T) = L_T^2(\rho_o, T_o) \cdot \left(\frac{\rho_o}{\rho}\right)^2 \cdot \left(\frac{T}{T_o}\right)^{m + \frac{1}{2}}$$ assuming To = T

$$L_{Ta} := L_T \cdot \left(\frac{1}{\rho_a}\right) = 2.838 \cdot cm \qquad \rho_a := \frac{1000 \cdot 1 + 10 \cdot 1.435}{1010} = 1.004$$

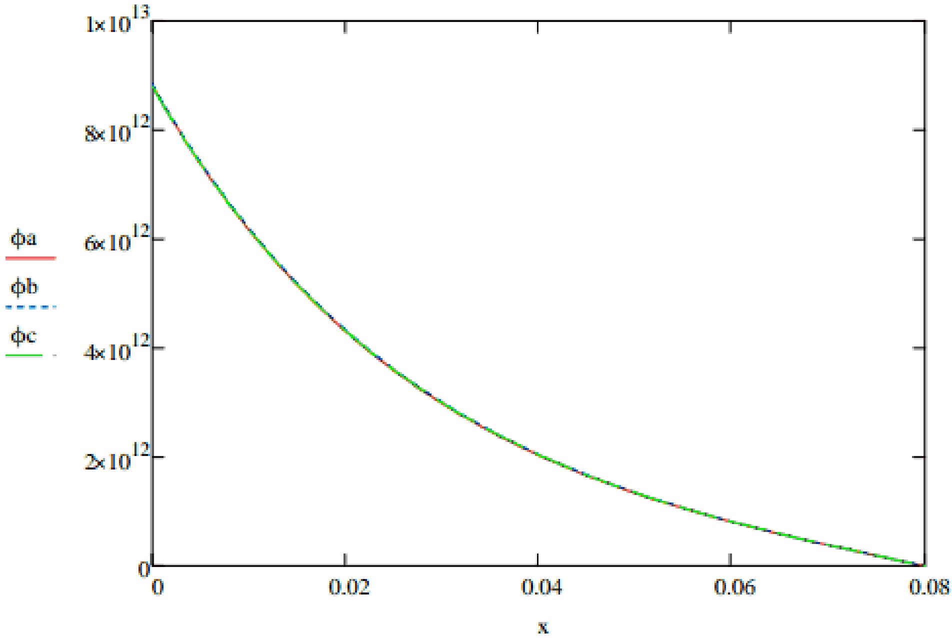
$$L_{Tb} := L_T \cdot \left(\frac{1}{\rho_b}\right) = 2.849 \cdot cm \qquad \rho_b := \frac{1000 \cdot 1 + 1 \cdot 1.435}{1001} = 1.00043$$

$$L_{Tc} := L_T \cdot \left(\frac{1}{\rho_c}\right) = 2.85 \cdot cm \qquad \rho_c := \frac{1000 \cdot 1 + 0.1 \cdot 1.435}{1000.1} = 1.00004$$

$$\phi a := \frac{S_1 \cdot L_{Ta}}{2 \cdot D} \cdot \left[\frac{\sinh\left[\frac{(a - x)}{L_{Ta}}\right]}{\cosh\left(\frac{a}{L_{Ta}}\right)} \right] \qquad \phi b := \frac{S_1 \cdot L_{Tb}}{2 \cdot D} \cdot \left[\frac{\sinh\left[\frac{(a - x)}{L_{Tb}}\right]}{\cosh\left(\frac{a}{L_{Tb}}\right)} \right]$$

$$\phi c := \frac{S_1 \cdot L_{Ta}}{2 \cdot D} \cdot \left[\frac{\sinh\left[\frac{(a - x)}{L_{Tc}}\right]}{\cosh\left(\frac{a}{L_{Tc}}\right)} \right]$$

This caluclation takes into account the change
in density due to adding boric acid to water. The small
concentrations result in a very small change in the flux.



Clearly this doesn't look like enough of an effect to make this problem worthwhile, I think the correct way to do this problem involves using the absorption cross-section for boric acid. I was unable to find this number.