PREV
Chapter 12

NEXT
Chapter 14

Section 13-2

13-1. a) Because factor df = total df – error df = 19 – 16 = 3 (and the degrees of freedom equals the number of levels minus one), 4 levels of the factor were used.

b) Because the total df = 19, there were 20 trials in the experiment. Because there are 4 levels for the factor, there were 5 replicates of each level.

c) From part (a), the factor df = 3

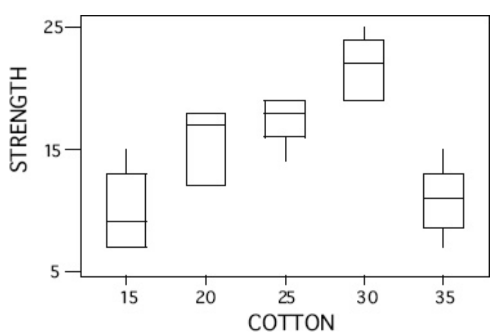
MS(Error) = 396.8/16 = 24.8, f = MS(Factor)/MS(Error) = 39.1/24.8 = 1.58.

From Appendix Table VI, 0.1 < P-value < 0.25

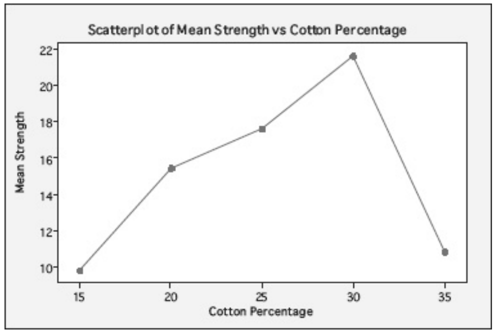
d) We fail to reject H_0. There are not significance differences in the factor level means at $\alpha = 0.05$.

COTTON	4	475.76	118.94	14.76	0.000
Error	20	161.20	8.06		
Total	24	636.96			

Reject H_0 and conclude that cotton percentage affects mean breaking strength.

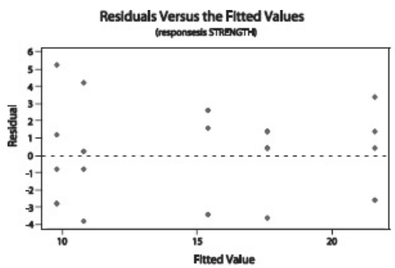


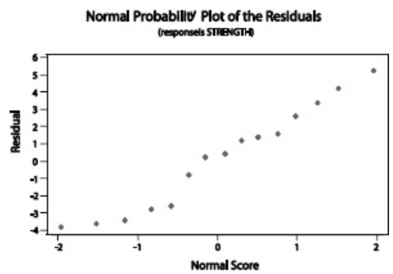
b) Tensile strength seems to increase up to 30% cotton and declines at 35% cotton.

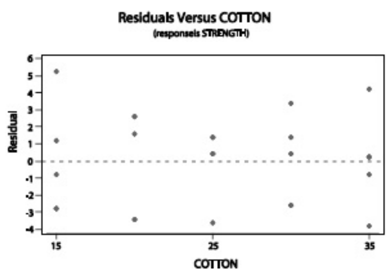



```
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level      N      Mean     StDev  ------+---------+---------+---------+
15         5     9.800     3.347  (-----*----)
20         5    15.400     3.130             (----*----)
25         5    17.600     2.074                 (----*----)
30         5    21.600     2.608                         (----*----)
35         5    10.800     2.864    (-----*----)
                                  ------+---------+---------+---------+
Pooled StDev =   2.839               10.0      15.0      20.0      25.0
```

c) The normal probability plot and the residual plots show that the model assumptions are reasonable.







13-5. a) Analysis of Variance for STRENGTH

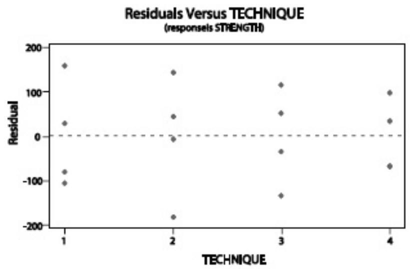
Source	DF	SS	MS	F	P
TECHNIQU	3	489740	163247	12.73	0.000
Error	12	153908	12826		

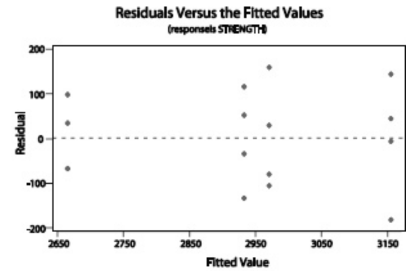
```
Total      15  643648
```

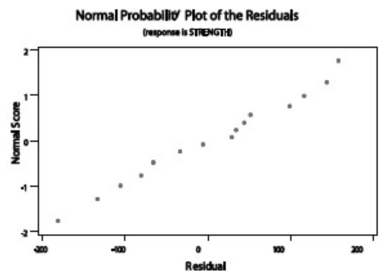
Reject H_0. Techniques affect the mean strength of the concrete.

b) *P-value* 0

c) Residuals are acceptable





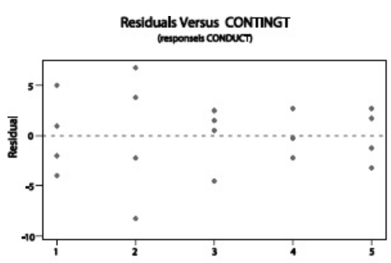


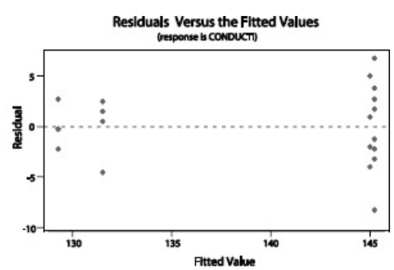
13-7. a) Analysis of Variance for CONDUCTIVITY

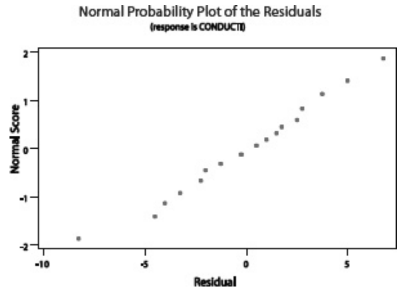
Source	DF	SS	MS	F	P
COATINGTYPE	4	1060.5	265.1	16.35	0.000
Error	15	243.3	16.2		
Total	19	1303.8			

Reject H_0, *P*-value 0

b) There is some indication of that the variability of the response may be increasing as the mean response increases. There appears to be an outlier on the normal probability plot.







c) 95% Confidence interval on the mean of coating type 1

$$\bar{y}_1 - t_{0.025,15}\sqrt{\frac{MS_E}{n}} \le \mu_i \le \bar{y}_1 + t_{0.015,15}\sqrt{\frac{MS_E}{n}}$$
$$145.00 - 2.131\sqrt{\frac{16.2}{4}} \le \mu_1 \le 145.00 + 2.131\sqrt{\frac{16.2}{4}}$$
$$140.71 \le \mu_1 \le 149.29$$

99% confidence interval on the difference between the means of coating types 1 and 4.

$$\bar{y}_1 - \bar{y}_4 - t_{0.005,15}\sqrt{\frac{2MS_E}{n}} \le \mu_1 - \mu_4 \le \bar{y}_1 - \bar{y}_4 + t_{0.005,15}\sqrt{\frac{2MS_E}{n}}$$
$$(145.00 - 129.25) - 2.947\sqrt{\frac{2(16.2)}{4}} \le \mu_1 - \mu_4 \le (145.00 - 129.25) - 2.947\sqrt{\frac{2(16.2)}{4}}$$
$$7.36 \le \mu_1 - \mu_4 \le 24.14$$

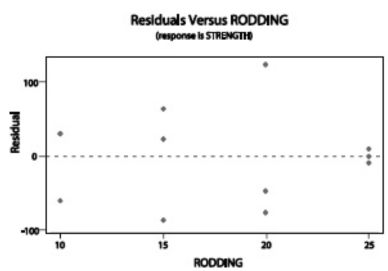
13-9. a) Analysis of Variance for STRENGTH

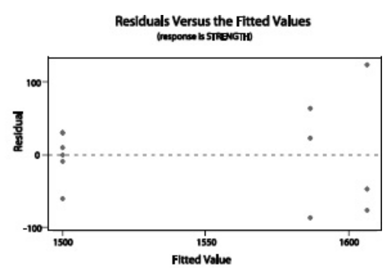
Source	DF	SS	MS	F	P
RODDING	3	28633	9544	1.87	0.214
Error	8	40933	5117		
Total	11	69567			

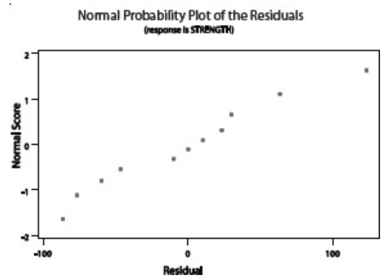
Fail to reject H_0

b) *P-value* = 0.214

c) The residual plot indicates some concern with nonconstant variance. The normal probability plot looks acceptable.







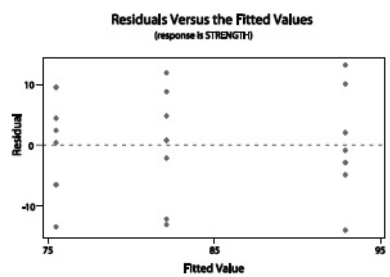
13-11. a) Analysis of Variance for STRENGTH

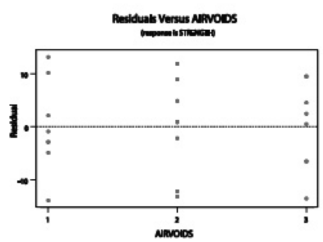
Source	DF	SS	MS	F	P
AIRVOIDS	2	1230.3	615.1	8.30	0.002
Error	21	1555.8	74.1		
Total	23	2786.0			

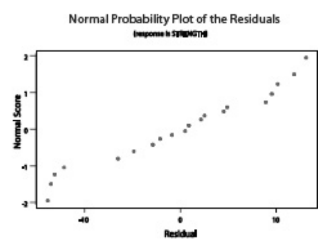
Reject H_0

b) P-value = 0.002

c) The residual plots indicate that the constant variance assumption is reasonable. The normal probability plot has some curvature in the tails but appears reasonable.







d) 95% Confidence interval on the mean of retained strength where there is a high level of air voids

$$\bar{y}_3 - t_{0.025,21}\sqrt{\frac{MS_E}{n}} \le \mu_i \le \bar{y}_3 + t_{0.015,21}\sqrt{\frac{MS_E}{n}}$$

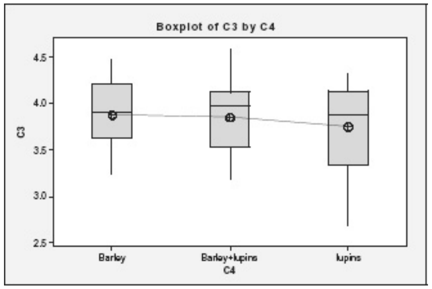
$$75.5 - 2.080\sqrt{\frac{74.1}{8}} \le \mu_3 \le 75.5 + 2.080\sqrt{\frac{74.1}{8}}$$

$$69.17 \le \mu_1 \le 81.83$$

e) 95% confidence interval on the difference between the means of retained strength at the high level and the low levels of air voids.

$$\bar{y}_1 - \bar{y}_3 - t_{0.025,21}\sqrt{\frac{2MS_E}{n}} \le \mu_1 - \mu_3 \le \bar{y}_1 - \bar{y}_3 + t_{0.025,21}\sqrt{\frac{2MS_E}{n}}$$
$$(92.875 - 75.5) - 2.080\sqrt{\frac{2(74.1)}{8}} \le \mu_1 - \mu_4 \le (92.875 - 75.5) + 2.080\sqrt{\frac{2(74.1)}{8}}$$
$$8.42 \le \mu_1 - \mu_4 \le 26.33$$

13-13. a) No, the diet does not affect the protein content of cow's milk. Comparative boxplots



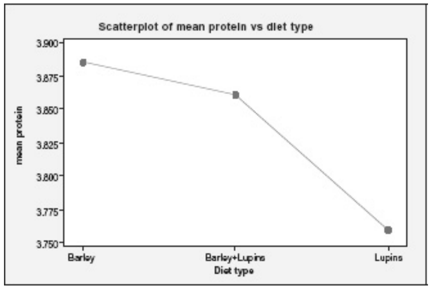
ANOVA

Source	DF	SS	MS	F	P
C4	2	0.235	0.118	0.72	0.489
Error	76	12.364	0.163		
Total	78	12.599			

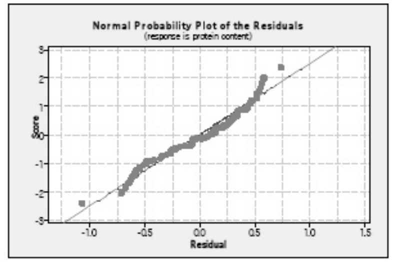
S = 0.4033   R–Sq = 1.87%   R–Sq(adj) = 0.00%

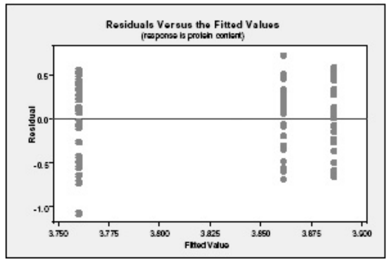
b) P-value = 0.489. The variability due to random error is SS_E = 0.146.

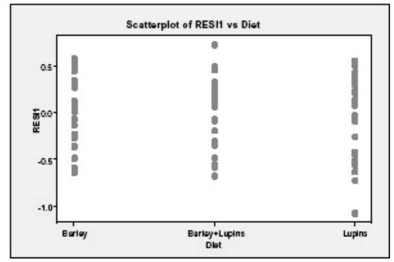
c) The Barley diet has the highest average protein content and lupins the lowest.



d) Based on the residual plots, there is no violation of the ANOVA assumptions.







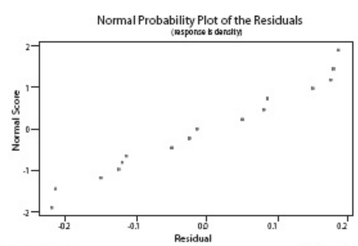
13-15. a) Analysis of Variance for TEMPERATURE

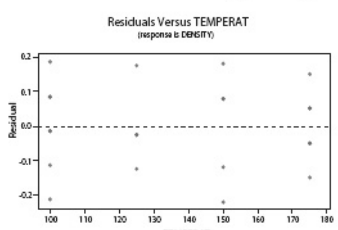
Source	DF	SS	MS	F	P
TEMPERAT	3	0.1391	0.0464	2.62	0.083
Error	18	0.3191	0.0177		
Total	21	0.4582			

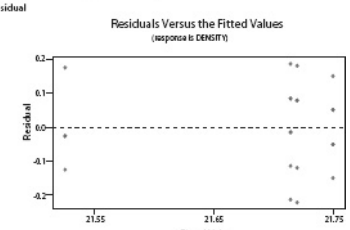
Fail to reject H_0

b) P-value = 0.083

c) Residuals are acceptable







13-17. Fisher's pairwise comparisons

```
Family error rate = 0.264
Individual error rate = 0.0500
Critical value = 2.086
Intervals for (column level mean) – (row level mean)
                15          20          25          30
      20      –9.346
              –1.854
      25     –11.546      –5.946
              –4.054       1.546
      30     –15.546      –9.946      –7.746
              –8.054      –2.454      –0.254
      35      –4.746       0.854       3.054       7.054
               2.746       8.346      10.546      14.546
```

Significant differences are detected between levels 15 and 20, 15 and 25, 15 and 30, 20 and 30, 20 and 35, 25 and 30, 25 and 35, and 30 and 35.

13-19. Fisher's pairwise comparisons

```
Family error rate = 0.184
Individual error rate = 0.0500
Critical value = 2.179
Intervals for (column level mean) – (row level mean)
                 1           2           3
       2        –360
                 –11
       3        –137          48
                 212         397
       4         130         316          93
                 479         664         442
```

Significance differences between levels 1 and 2, 1 and 4, 2 and 3, 2 and 4, and 3 and 4.

13-21. Fisher's pairwise comparisons

```
Family error rate = 0.0649
Individual error rate = 0.0100
Critical value = 2.947
Intervals for (column level mean) – (row level mean)

                 1           2           3           4
       2      –8.642
               8.142
       3       5.108       5.358
              21.892      22.142
```

```
4       7.358      7.608     -6.142
       24.142     24.392     10.642
5      -8.642     -8.392    -22.142    -24.392
        8.142      8.392     -5.358     -7.608
```

Significant differences between 1 and 3, 1 and 4, 2 and 3, 2 and 4, 3 and 5, 4 and 5.

13-23. Fisher's pairwise comparisons

```
Family error rate = 0.118
Individual error rate = 0.0500
Critical value = 2.080
Intervals for (column level mean) – (row level mean)
                 1          2
2       1.799
       19.701
3       8.424     -2.326
       26.326     15.576
```

Significant differences between levels 1 and 2; and 1 and 3.

13-25. a) There is no significant difference in protein content between the three diet types.

```
Fisher 99% Individual Confidence Intervals
All Pairwise Comparisons among Levels of C4

Simultaneous confidence level = 97.33%


C4 = Barley subtracted from:

C4                Lower   Center   Upper
Barley+lupins  -0.3207  -0.0249  0.2709
lupins         -0.4218  -0.1260  0.1698

C4             -------+---------+---------+---------+--
Barley+lupins      (-----------*-----------)
lupins         (-----------*-----------)
               -------+---------+---------+---------+--
                   -0.25      0.00      0.25      0.50

C4 = Barley+lupins subtracted from:

C4       Lower   Center   Upper  -------+---------+---------+---------+--
lupins  -0.3911  -0.1011  0.1889   (-----------*-----------)
                                 -------+---------+---------+---------+--
                                     -0.25      0.00      0.25      0.50
```

b) The mean values are: 3.886, 3.8611, 3.76 (barley, b+l, lupins)

From the ANOVA the estimate of σ can be obtained

Source	DF	SS	MS	F	P
C4	2	0.235	0.118	0.72	0.489
Error	76	12.364	0.163		
Total	78	12.599			

```
S = 0.4033   R–Sq = 1.87%   R–Sq(adj) = 0.00%
The minimum sample size could be used to calculate the standard error of a
sample mean
```

$$\hat{\sigma}_{\bar{X}} = \sqrt{\frac{MS_E}{b}} = \sqrt{\frac{0.163}{25}} = 0.081$$

The graph would not show any differences between the diets.

13-27. $\overline{\mu} = 188$, $\tau_1 = -13$, $\tau_2 = 2$, $\tau_3 = -28$, $\tau_4 = 12$, $\tau_5 = 27$.

$$\Phi^2 = \frac{n(1830)}{5(100)} = n, \qquad a-1 = 4 \qquad a(n-1) = 5(n-1)$$

Various choices for n yield:

n	Φ^2	Φ	a(n – 1)	Power = 1 – β
2	7.32	2.7	5	0.55
3	10.98	3.13	10	0.95

Therefore, $n = 3$ is needed.

Section 13-3

13-29. a) Analysis of Variance for OUTPUT

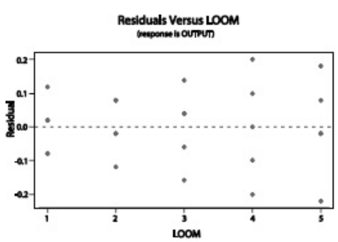
```
Source     DF       SS       MS       F       P
LOOM        4   0.3416   0.0854    5.77   0.003
Error      20   0.2960   0.0148
Total      24   0.6376
```

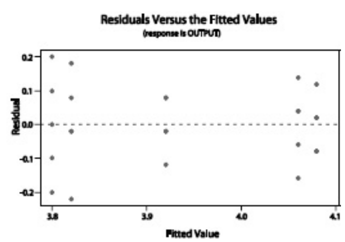
Reject H_0; there are significant differences among the looms.

b) $$\hat{\sigma}_\tau^2 = \frac{MS_{Treatments} - MS_E}{n} = \frac{0.0854 - 0.0148}{5} = 0.01412$$

c) $\hat{\sigma}^2 = MS_E = 0.0148$

d) Residuals are acceptable





13-31. a) Analysis of Variance for BRIGHTNENESS

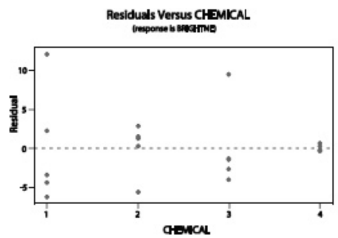
Source	DF	SS	MS	F	P
CHEMICAL	3	54.0	18.0	0.75	0.538
Error	16	384.0	24.0		
Total	19	438.0			

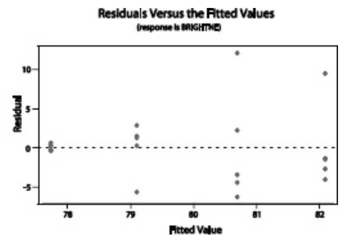
Fail to reject H_0; there is no significant difference among the chemical types.

b) $\hat{\sigma}_\tau^2 = \frac{18.0 - 24.0}{5} = -1.2$ set equal to 0

c) $\hat{\sigma}^2$ = 24.0

d) Variability is smaller in chemical 4. There is some curvature in the normal probability plot.





13-33. a) Instead of testing the hypothesis that the individual treatment effects are zero, we are testing whether there is variability in protein content between all diets.

$$H_0 : \sigma_\tau^2 = 0$$
$$H_1 : \sigma_\tau^2 \neq 0$$

b) The statistical model is

$$y = \mu + \tau_i + \varepsilon_{ij} \begin{cases} i = 1,2,\ldots,a \\ j = 1,2,\ldots,n \end{cases}$$

$$\varepsilon_i \sim N(0,\sigma^2) \text{ and } \tau_i \sim N(0,\sigma_\tau^2)$$

c) The **last TWO observations were omitted** from two diets to generate equal sample sizes with n = 25.

ANOVA: Protein versus DietType

Analysis of Variance for Protein

Source	DF	SS	MS	F	P
DietType	2	0.2689	0.1345	0.82	0.445
Error	72	11.8169	0.1641		
Total	74	12.0858			

S = 0.405122   R-Sq = 2.23%   R-Sq(adj) = 0.00%

$\sigma^2 = MS_E = 0.1641$

$$\sigma_\tau^2 = \frac{MS_\tau - MS_E}{n} = \frac{0.1345 - 0.1641}{25} = -0.001184$$

Section 13-4

13-35. The output from Minitab follows.

Source	DF	SS	MS	F	P
Factor	2	1952.64	976.322	147.35	0.000
Block	11	198.54	18.049	2.72	0.022
Error	22	145.77	6.626		
Total	35	2296.95			

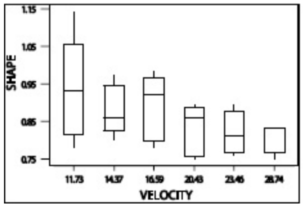
S = 2.574   R–Sq = 93.65%   R–Sq(adj) = 89.90%

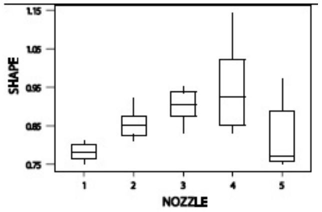
Because the P-value for the factor is near zero there are significant differences in the factor level means at α = 0.05 or α = 0.01.

13-37. a) Analysis of Variance for SHAPE

Source	DF	SS	MS	F	P
NOZZLE	4	0.102180	0.025545	8.92	0.000
VELOCITY	5	0.062867	0.012573	4.39	0.007
Error	20	0.057300	0.002865		
Total	29	0.222347			

Reject H_0; nozzle type affects shape measurement.





b) Fisher's pairwise comparisons

Family error rate = 0.268

Individual error rate = 0.0500

Critical value = 2.060

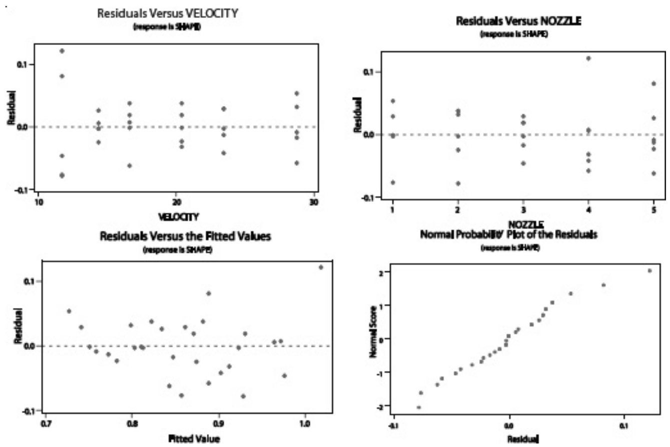
Intervals for (column level mean) – (row level mean)

	1	2	3	4
2	−0.15412			
	0.01079			
3	−0.20246	−0.13079		
	−0.03754	0.03412		
4	−0.24412	−0.17246	−0.12412	
	−0.07921	−0.00754	0.04079	

5	−0.11412	−0.04246	0.00588	0.04754
	0.05079	0.12246	0.17079	0.21246

There are significant differences between levels 1 and 3; 4; 2 and 4; 3 and 5; and 4 and 5.

c) The residual analysis shows that there is some inequality of variance. The normal probability plot is acceptable.

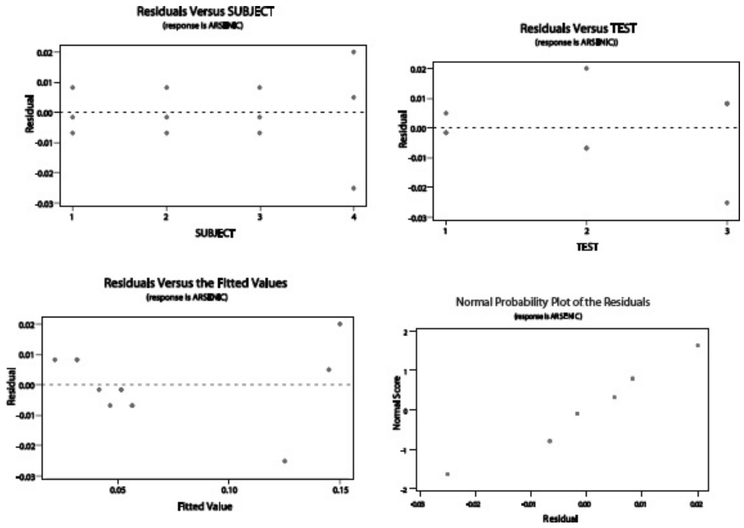


13-39. a) Analysis of Variance for ARSENIC

Source	DF	SS	MS	F	P
TEST	2	0.0014000	0.0007000	3.00	0.125
SUBJECT	3	0.0212250	0.0070750	30.32	0.001
Error	6	0.0014000	0.0002333		
Total	11	0.0240250			

Fail to reject H_0; there is no evidence of differences between the tests.

b) Some indication of variability increasing with the magnitude of the response.



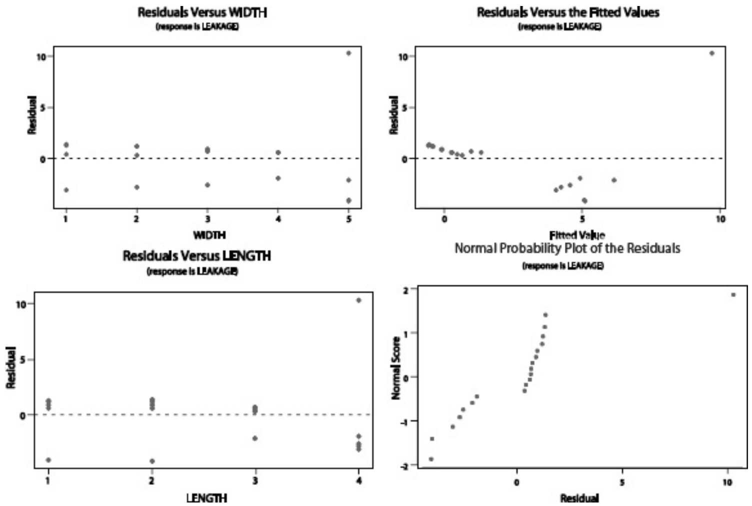
13-41. A version of the electronic data file has the reading for length 4 and width 5 as 2. It should be 20.

a) Analysis of Variance for LEAKAGE

Source	DF	SS	MS	F	P
LENGTH	3	72.66	24.22	1.61	0.240
WIDTH	4	90.52	22.63	1.50	0.263
Error	12	180.83	15.07		
Total	19	344.01			

Fail to reject H_0, mean leakage voltage does not depend on the channel length.

b) One unusual observation in width 5, length 4. There are some problems with the normal probability plot, including the unusual observation.



c) Analysis of Variance for LEAKAGE VOLTAGE

Source	DF	SS	MS	F	P
LENGTH	3	8.1775	2.7258	6.16	0.009
WIDTH	4	6.8380	1.7095	3.86	0.031
Error	12	5.3100	0.4425		
Total	19	20.3255			

Reject H_0. And conclude that the mean leakage voltage does depend on channel length. By removing the data point that was erroneous, the analysis results in a conclusion. The erroneous data point that was an obvious outlier had a strong effect the results of the experiment.

13-43. a) Because MS = SS/df(Factor), df(Factor) = SS/MS = 126.880/63.4401 = 2. The number of levels = df(Factor) + 1 = 2 + 1 = 3. Therefore, 3 levels of the factor were used.

b) Because df(Total) = df(Factor) + df(Block) + df(Error)

11 = 3 + df(Block) + 6. Therefore, df(Block) = 2. Therefore, 3 blocks were used in the experiment.

c) From parts (a) and (b), df(Factor) = 3 and df(Block) = 2

SS(Error) = df(Error)MS(Error) = (6)2.7403 = 16.4418

F = MS(Factor)/MS(Error) = 63.4401/2.7403 = 23.15

From Appendix Table VI, P-value < 0.01

d) Because the P-value < 0.01 we reject H_0. There are significant differences in the factor level means at $\alpha = 0.05$ or $\alpha = 0.01$.

Supplemental Exercises

13-45. a) Analysis of Variance for RESISTANCE

Source	DF	SS	MS	F	P
ALLOY	2	10941.8	5470.9	76.09	0.000
Error	27	1941.4	71.9		
Total	29	12883.2			

Reject H_0; the type of alloy has a significant effect on mean contact resistance.

b) Fisher's pairwise comparisons

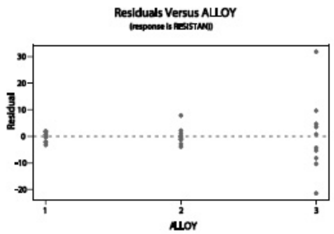
```
Family error rate = 0.119
Individual error rate = 0.0500
Critical value = 2.052
Intervals for (column level mean) – (row level mean)
               1           2
    2      –13.58
             1.98
    3      –50.88      –45.08
           –35.32     –29.52
```

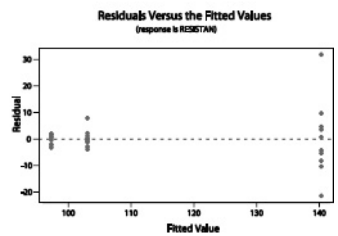
There are differences in the mean resistance for alloy types 1 and 3; and types 2 and 3.

c) 99% confidence interval on the mean contact resistance for alloy 3

$$\bar{y}_3 - t_{0.005,27}\sqrt{\frac{MS_E}{n}} \le \mu_i \le \bar{y}_3 + t_{0.005,27}\sqrt{\frac{MS_E}{n}}$$
$$140.4 - 2.771\sqrt{\frac{71.9}{10}} \le \mu_3 \le 140.4 + 2.771\sqrt{\frac{71.9}{10}}$$
$$132.97 \le \mu_1 \le 147.83$$

d) Variability of the residuals increases with the response. The normal probability plot has some curvature in the tails, indicating a problem with the normality assumption. A transformation of the response should be conducted.





13-47. a) Analysis of Variance for VOLUME

Source	DF	SS	MS	F	P
TEMPERATURE	2	16480	8240	7.84	0.007
Error	12	12610	1051		

```
Total         14     29090
```

Reject H_0.

b) *P-value* = 0.007

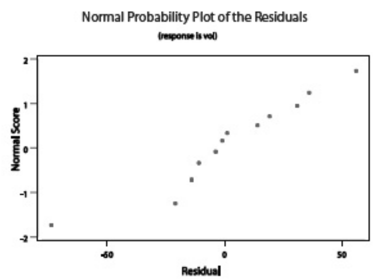
c) Fisher's pairwise comparisons

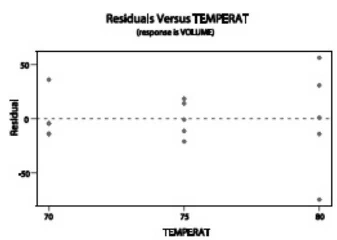
```
Family error rate = 0.116
Individual error rate = 0.0500
Critical value = 2.179
Intervals for (column level mean) – (row level mean)
```

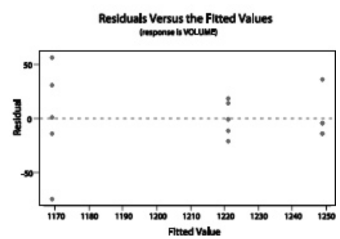
	70	75
75	−16.7	
	72.7	
80	35.3	7.3
	124.7	96.7

There are significant differences in the mean volume for temperature levels 70 and 80; and 75 and 80. The highest temperature results in the smallest mean volume.

d) There are some relatively small differences in the variability at the different levels of temperature. The variability decreases with the fitted values. There is an unusual observation on the normal probability plot.







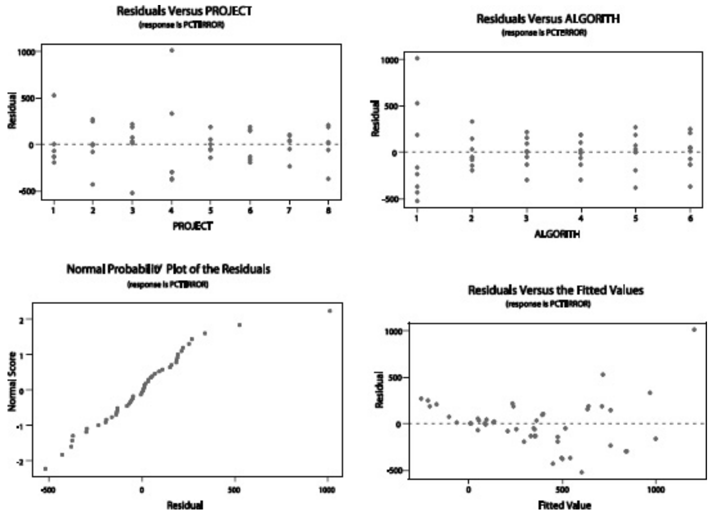
13-49. a) Analysis of Variance for PCTERROR

Source	DF	SS	MS	F	P
ALGORITH	5	2825746	565149	6.23	0.000
PROJECT	7	2710323	387189	4.27	0.002

Error	35	3175290	90723
Total	47	8711358	

Reject H_0; the algorithms are significantly different.

b) The residuals look acceptable, except there is one unusual point.



c) The best choice is algorithm 5 because it has the smallest mean and a low variability.

13-51. a) $\mu = 1.6$, $\Phi^2 = 0.284$, $\Phi = 0.5333$

Numerator degrees of freedom = $a-1 = 4 = v_1$

Denominator degrees of freedom = $a(n-1) = 15 = v_2$

From Chart Figure 13-6, $\beta \approx 0.8$ and the power = $1 - \beta = 0.2$

b)

n	Φ^2	Φ	a(n − 1)	β	Power = 1 − β
50	3.56	1.89	245	0.0	0.95

The sample size should be approximately $n = 50$.

Mind Expanding Exercises

13-53. $MS_E = \dfrac{\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \overline{y}_i)^2}{a(n-1)}$ and $y_{ij} = \mu + a_i + \varepsilon_{ij}$. Then $y_{ij} - \overline{y}_i = \varepsilon_{ij} - \overline{\varepsilon}_{i.}$ and

$\dfrac{\sum_{j=1}^{n}(\varepsilon_{ij} - \overline{\varepsilon}_{i.})}{n-1}$ is recognized to be the sample variance of the independent random variables $\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{in}$.

Therefore, $E=\left[\frac{\sum_{j=1}^{n}(\varepsilon_{ij}-\bar{\varepsilon}_{i.})^2}{n-1}\right]=\sigma^2$ and $E(MS_E)=\sum_{i=1}^{a}\frac{\sigma^2}{a}=\sigma^2$.

The development would not change if the random effects model had been specified because $y_{ij}-\bar{y}_{i}=\varepsilon_{ij}-\bar{\varepsilon}_{i.}$ for this model also.

13-55. $MS_E=\frac{\sum_{i=1}^{2}\sum_{j=1}^{n}(y_{ij}-\bar{y}_{i.})^2}{2(n-1)}$ and $\frac{\sum_{j=1}^{n}(y_{ij}-\bar{y}_{i.})^2}{n-1}$ is recognized as the sample standard deviation calculated from

the data from population i. Then, $MS_E=\frac{s_1^2+s_2^2}{2}$ which is the pooled variance estimate used in the t-test.

13-57. If b, c, and d are the coefficients of three orthogonal contrasts, it can be shown that

$\frac{(\sum_{i=1}^{4}b_iy_{i.})^2}{\sum_{i=1}^{a}b_i^2}+\frac{(\sum_{i=1}^{a}c_iy_{i.})^2}{\sum_{i=1}^{a}c_i^2}+\frac{(\sum_{i=1}^{a}d_iy_{i.})^2}{\sum_{i=1}^{a}d_i^2}=\sum_{i=1}^{a}y_{i.}^2-\frac{(\sum_{i=1}^{a}y_{i.})^2}{a}$ always holds. Upon dividing both sides by n,

we have $Q_1^2+Q_2^2+Q_3^2=\sum_{i=1}^{a}\frac{y_{i}^2}{n}-\frac{y_{..}^2}{N}$ which equals $SS_{treatments}$. The equation above can be obtained from a

geometrical argument. The square of the distance of any point in four-dimensional space from the zero point can be expressed as the sum of the squared distance along four orthogonal axes. Let one of the axes be the 45 degree line and let the point be ($y_{1.}$, $y_{2.}$, $y_{3.}$, $y_{4.}$). The three orthogonal contrasts are the other three axes. The square of the

distance of the point from the origin is $\sum_{i=1}^{a}y_{i.}^2$ and this equals the sum of the squared distances along each of the

four axes.

13-59. $MS_E=\frac{\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij}-\bar{y}_{i.})^2}{a(n-1)}=\frac{\sum_{i=1}^{a}s_i^2}{a}$ where $s_i^2=\frac{\sum_{j=1}^{n}(y_{ij}-\bar{y}_{i.})^2}{n-1}$. Because s_i^2 is the sample variance of

$y_{i1},y_{i2},\ldots,y_{in}$, $\frac{(n-1)S_i^2}{\sigma^2}$ has a chi-square distribution with n – 1 degrees of freedom. Then, $\frac{a(n-1)MS_E}{\sigma^2}$ is a sum of independent chi-square random variables. Consequently, $\frac{a(n-1)MS_E}{\sigma^2}$ has a chi-square distribution with

a(n – 1) degrees of freedom. Consequently,

$$P(\chi^2_{1-\frac{\alpha}{2},a(n-1)} \le \frac{a(n-1)MS_E}{\sigma^2} \le \chi^2_{\frac{\alpha}{2},a(n-1)}) = 1-\alpha$$
$$= P\left(\frac{a(n-1)MS_E}{\chi^2_{\frac{\alpha}{2},a(n-1)}} \le \sigma^2 \le \frac{a(n-1)MS_E}{\chi^2_{1-\frac{\alpha}{2},a(n-1)}}\right)$$

Using the fact that a(n – 1) = N – a completes the derivation.

13-61. a) As in Exercise 13-54, $\frac{MS_T}{MS_E}\frac{\sigma^2}{n\sigma_\tau^2+\sigma^2}$ has an $F_{(a-1),(N-a)}$ distribution.

and

$$1-\alpha = P(L \le \frac{\sigma_\tau^2}{\sigma^2} \le U)$$
$$= P(\frac{1}{U} \le \frac{\sigma^2}{\sigma_\tau^2} \le \frac{1}{L})$$
$$= P(\frac{1}{U}+1 \le \frac{\sigma^2}{\sigma_\tau^2}+1 \le \frac{1}{L}+1)$$
$$= P(\frac{L}{L+1} \le \frac{\sigma_\tau^2}{\sigma^2+\sigma_\tau^2} \le \frac{U}{U+1})$$

b) $$1-\alpha = P(L \le \frac{\sigma_\tau^2}{\sigma^2} \le U)$$
$$= P(L+1 \le \frac{\sigma_\tau^2+1}{\sigma^2} \le U+1)$$
$$= P(L+1 \le \frac{\sigma_\tau^2+\sigma^2}{\sigma^2} \le U+1)$$
$$= P(\frac{1}{U+1} \le \frac{\sigma^2}{\sigma^2+\sigma_\tau^2} \le \frac{1}{L+1})$$

Therefore, $(\frac{1}{U+1},\frac{1}{L+1})$ is a confidence interval for $\frac{\sigma^2}{\sigma_\tau^2+\sigma^2}$

13-63. a) If *A* is the accuracy of the interval, then $t_{\frac{\alpha}{2},a(n-1)}\sqrt{\frac{2MS_E}{n}} = A$

Squaring both sides yields $t^2_{\frac{\alpha}{2},a(n-1)}\frac{2MS_E}{n} = A^2$

As in Exercise 13-48, $t^2_{\frac{\alpha}{2},a(n-1)} = F_{\alpha,1,a(n-1)}$. Then,

$$n = \frac{2MS_E F_{\alpha,1,a(n-1)}}{A^2}.$$

b) Because n determines one of the degrees of freedom of the tabulated F value on the right-side of the equation in part (a), some approximation is needed. Because the value for a 95% confidence interval based on a normal distribution is 1.96, we approximate $t_{\frac{\alpha}{2},a(n-1)}$ by 2 and we approximate

$t^2_{\frac{\alpha}{2},a(n-1)} = F_{\alpha,1,a(n-1)}$ by 4.

Then, $n = \frac{2(4)(4)}{4} = 8$. With n = 8, $a(n-1)$ = 35 and $F_{0.05,1,35}$ = 4.12.

The value 4.12 can be used for F in the equation for n and a new value can be computed for n as

$$n = \frac{2(4)(4.12)}{4} = 8.24 \cong 8.$$

Because the solution for n did not change, we can use $n = 8$. If needed, another iteration could be used to refine the value of n.

PREV
Chapter 12

NEXT
Chapter 14