전자 회로 1
Lecture 2 (Op-Amp I)

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이 강의 노트는 전자공학부 곽노준 교수께서 08.03에 작성한 것으로 노트제공에 감사드림.

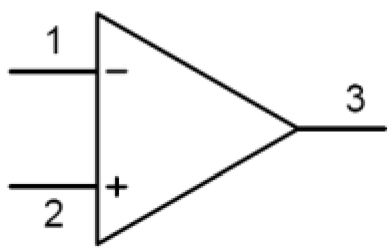
Overview

- Reading:
  - Sedra & Smith Chapter 2.1~2.4
  - Chap. 2.4.2 is omitted in this lecture. (Self study needed)

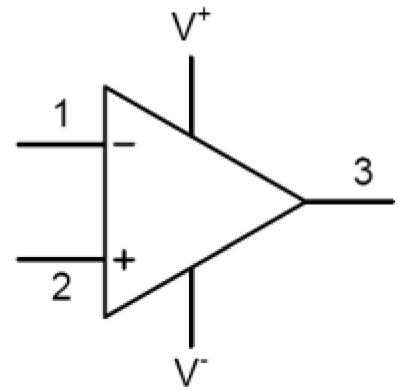
- Outline
  - Ideal Op-Amp
  - Inverting/non-inverting configuration
  - Difference Amp.

OP AMP

- OP AMP = Operational Amplifier (연산증폭기)
  - + / - / 미분 / 적분 등의 연산이 가능
- Symbols



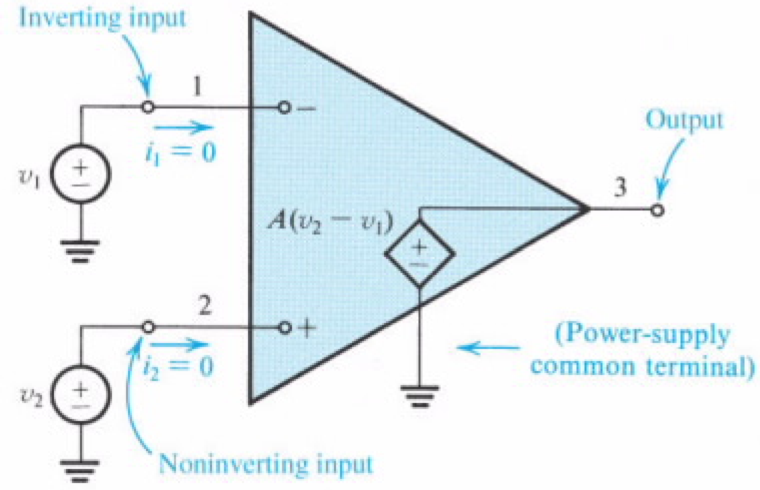
op amp symbol
(we will use most often)



op amp symbol
with power supply connections

  - 최소한 3개의 터미널이 있음 (2 input / 1 output)
  - DC power도 필요 (1개 혹은 2개: V+ / V-)

Ideal Op-Amp



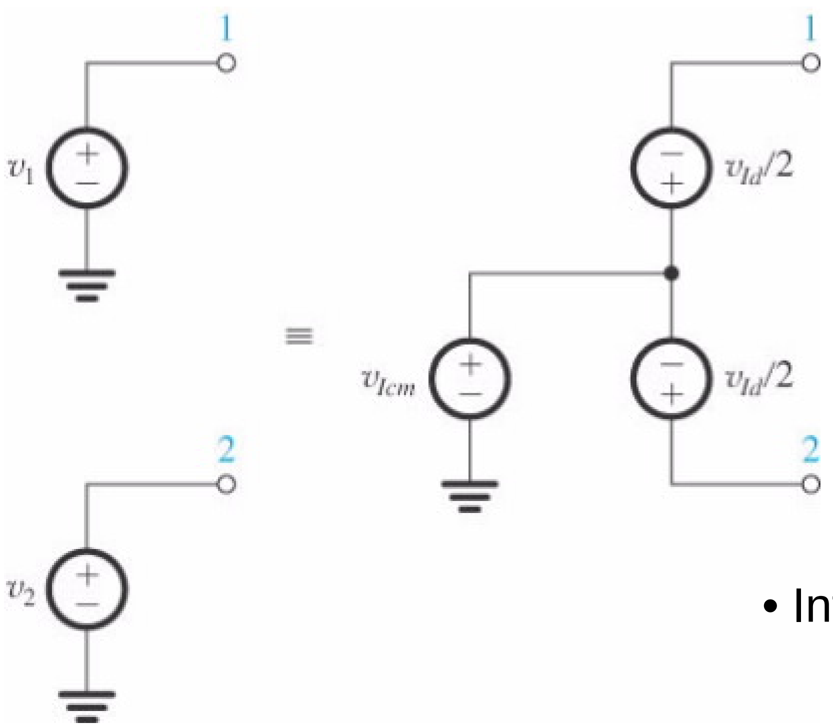
- OP-AMP는 input signal의 차이 **(v2-v1)**를 증폭해서 output에 나타낸다.
- 즉 v0 = A (v2-v1): voltage amplifier
- A: differential gain
  open-loop gain

TABLE 2.1 Characteristic of the ideal Op Amp

1. Infinite input impedance
2. Zero output impedance
3. Zero common-mode gain or, equivalently, infinite common-mode rejection
4. Infinite open-loop gain A
5. Infinite bandwidth
6. Ideal voltage controlled voltage source

Common & differential mode signals

- Differential input signal: Id 21 **(2.1)**
- Common-mode input signal: Icm $\frac{1}{2}()$ 12 **(2.2)**

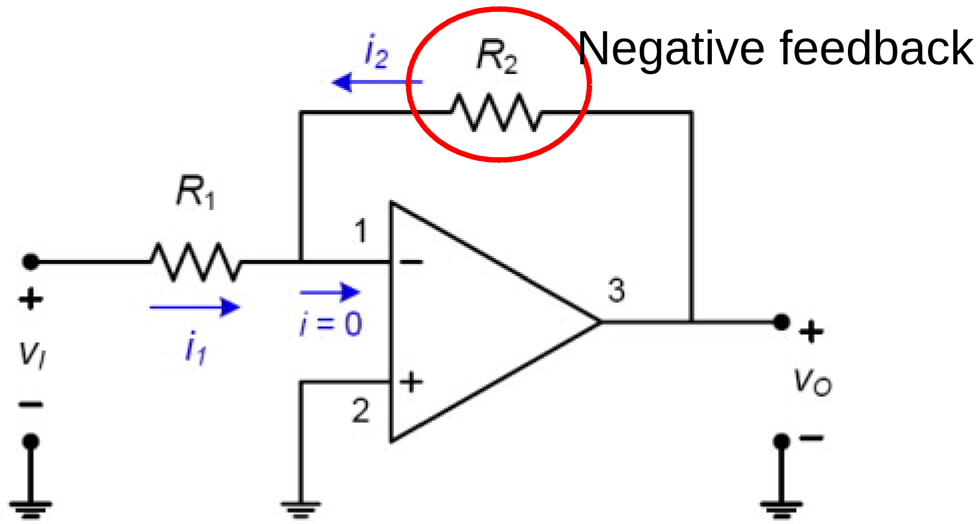


1 $IcmId$ $/2$ **(2.3)**

2 $IcmId$ $/2$ **(2.4)**

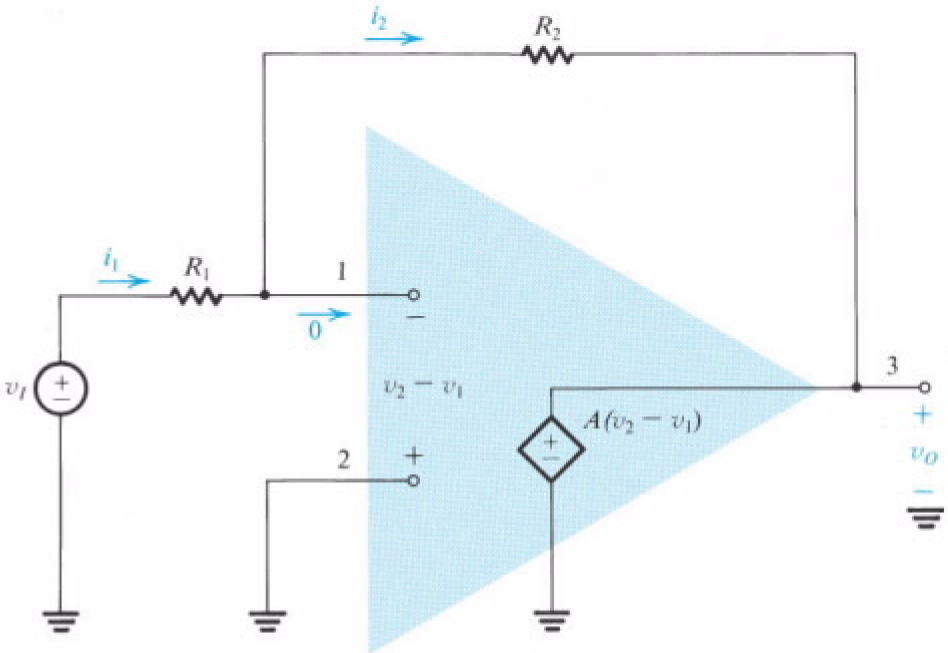
- Infinite Common-mode rejection:
  v1과 v2에 공통으로 있는 성분을 전혀 증폭하지 않는다.

Inverting configuration (1)

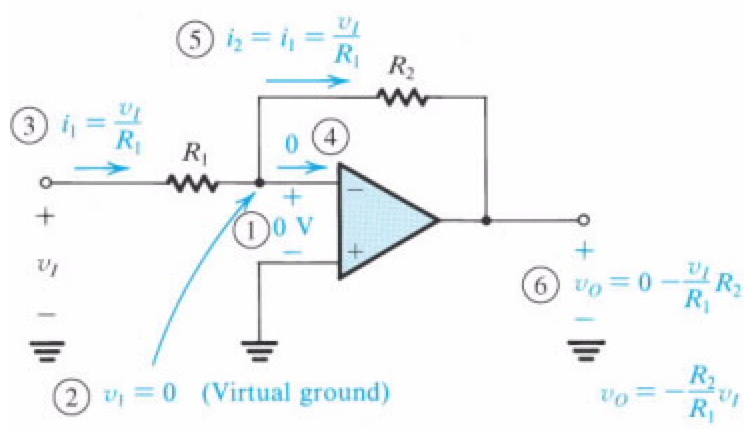


- Closed-loop gain G=Vo/Vi
- A가 무한대라고 가정하면, V1-V2 = Vo/A = 0
  - Virtual short circuit
  - V2 = 0 → V1 = 0 이므로 V1을 virtual ground라고도 함.

Inverting configuration (2)



(a)



(b)

01122 iRR 0 $\frac{I}{R_I}$

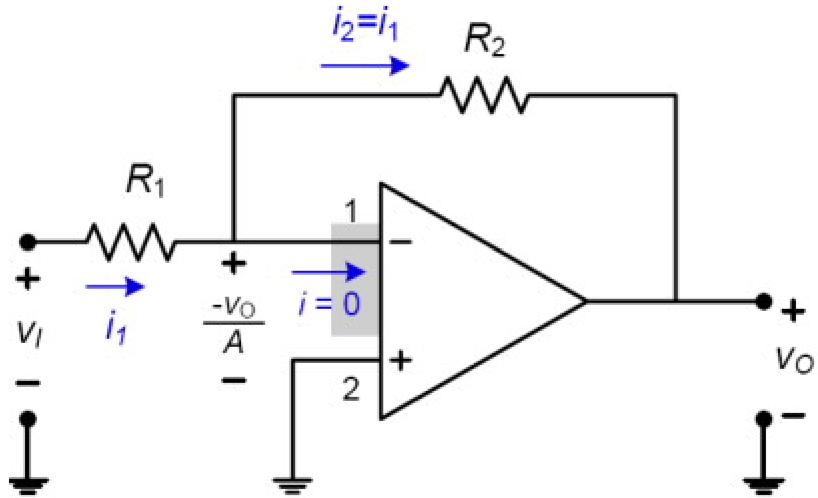
$i_1 \quad \frac{III \quad 1}{RRR_{III}} \quad \frac{\quad 0 \quad}{} \quad \frac{}{}$

$$G = \frac{0}{I} \quad \frac{R_2}{R_1}$$

- R1과 R2의 비율을 변화시킴으로써 closed-loop gain G를 변화시킬 수 있다. (G는 A와 independent; if A is infinite)

Finite open-loop gain

- A를 무한대로 만드는 것은 물리적으로 불가능
- What if A is finite?
- Virtual ground 대신 terminal 의 전압이 –Vo/A라고 가정



$$G \equiv \frac{v_O}{v_1} = \frac{-R_2/R_1}{1 + (1 + R_2/R_1)/A}$$

i_1 II (/A)/A 00 RR II

0122 000 AA iRR I /A R_1

- A→infinity ➔ G→-R2/R1

  V1→0 ➔ Virtual ground 성립
- Open loop gain A의 영향을 줄이기 위해

$$1 + \frac{R_2}{R_1} \ll A$$

$$v_O\left(1 + \frac{1}{A} + \frac{R_2}{R_1 A}\right) = -\frac{R_2}{R_1} v_I$$

Input resistance (closed-loop)

- Ideal op-amp를 가정하면 (A= infinity) input resistance:

  $$R_I R \quad \frac{II}{i_I R} \quad \frac{}{I\,/} \quad 1$$

- What if A = finite?
  - solve $R_{in} = v_I / i_1$

$$i_1 = \frac{v_I + v_O/A}{R_1} \quad \text{and} \quad v_O = -\frac{i_1 R_2}{1 + 1/A}$$

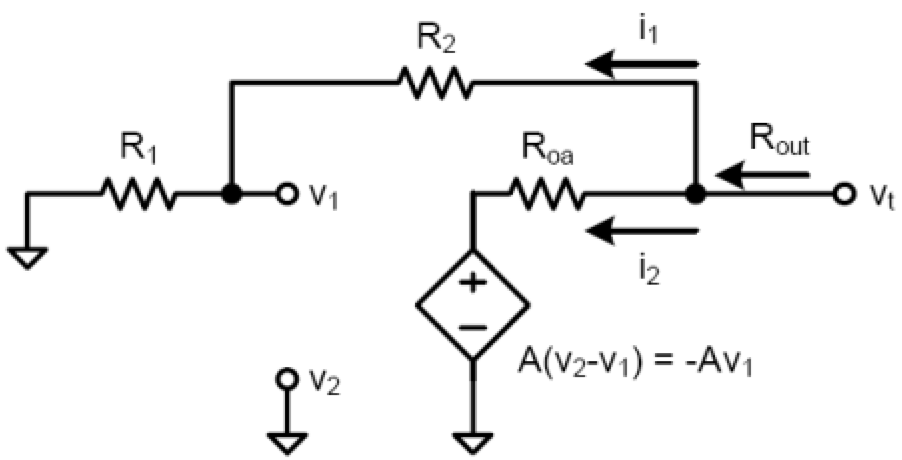
$$\begin{aligned} i_1 R_1 &= v_I - \frac{i_1 R_2}{A+1} \\ \frac{v_I}{i_1} &= R_1 + \frac{R_2}{A+1} \\ R_{in} &= R_1 + \frac{R_2}{A+1} \end{aligned}$$

High gain G를 얻기 위해서는 R1이 작아져야 한다. (R2를 크게 할 수는 없기 때문에)

➔small input resistance problem
(solution in Example 2.2)

Output resistance

- Output resistance를 구하기 위해서는
  - Input voltage를 0으로 하고 강제로 output에 전압을 준 후 Vo/Io를 구한다.



$$i_1 = \frac{v_t}{R_2 + R_1} \quad \text{and} \quad v_1 = \frac{R_1}{R_1 + R_2} v_t$$

$$i_2 = \frac{v_t - (-Av_1)}{R_{oa}} = \frac{v_t\left(1 + A\frac{R_1}{R_1+R_2}\right)}{R_{oa}}$$

$$R = \frac{v_t}{i_2} = \frac{R_{oa}}{1 + A\frac{R_1}{R_1+R_2}}$$

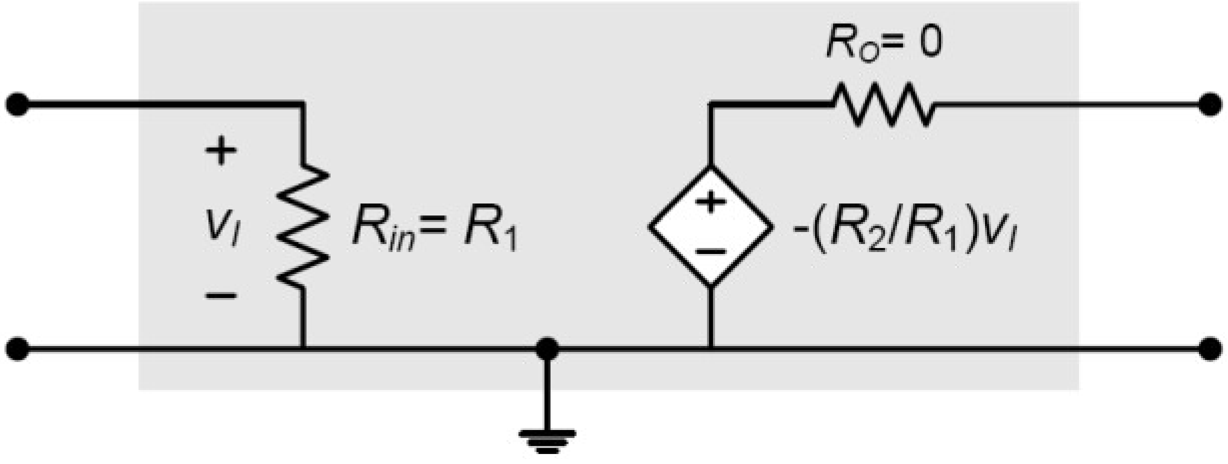
$$R_{out} = R \| (R_1 + R_2)$$

$$R_{out} \approx \frac{R_{oa}}{1 + A\frac{R_1}{R_1+R_2}}$$

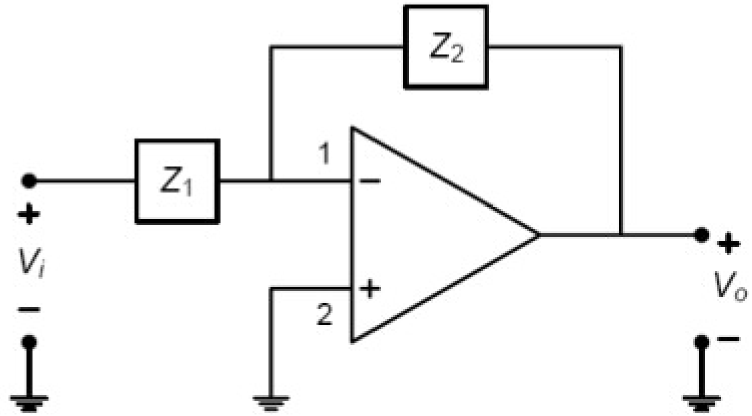
- 그림 2.6(a)에서는 $R_{oa} = 0$ → 작은 output resistance (Good!)

Model of inverting configuration

- Closed-loop inverting configuration은 다음과 같은 voltage controlled voltage source (voltage amplifier) 로 모델이 가능

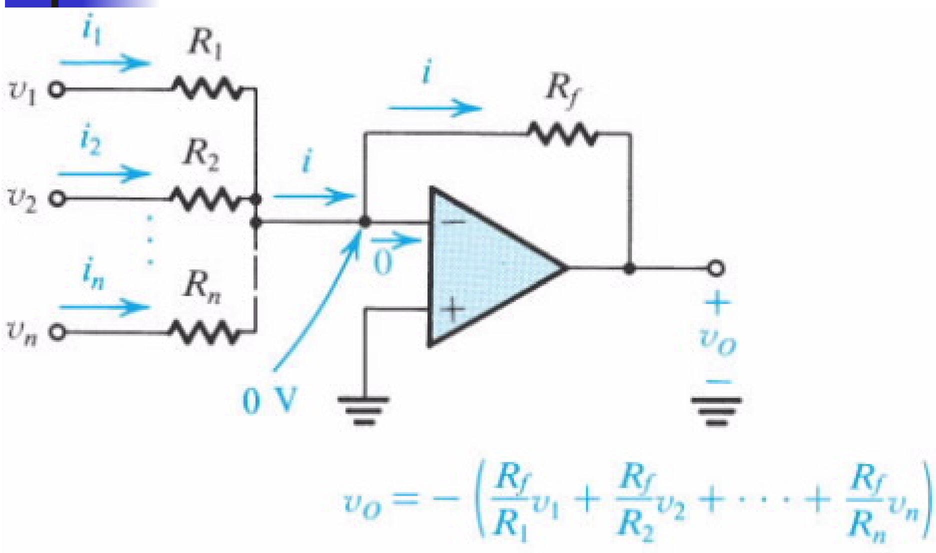


Inverting config. with general impedance



- R1, R2 → Z1, Z2로 대체 $\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$
- Z1, Z2를 바꿔가면서 다음을 만들 수 있다.
  - Integrator (Chap. 2.8)
  - Differentiator (Chap. 2.8)
  - Summer (Chap. 2.2.4)
  - …

Examples: The Weighted Summer

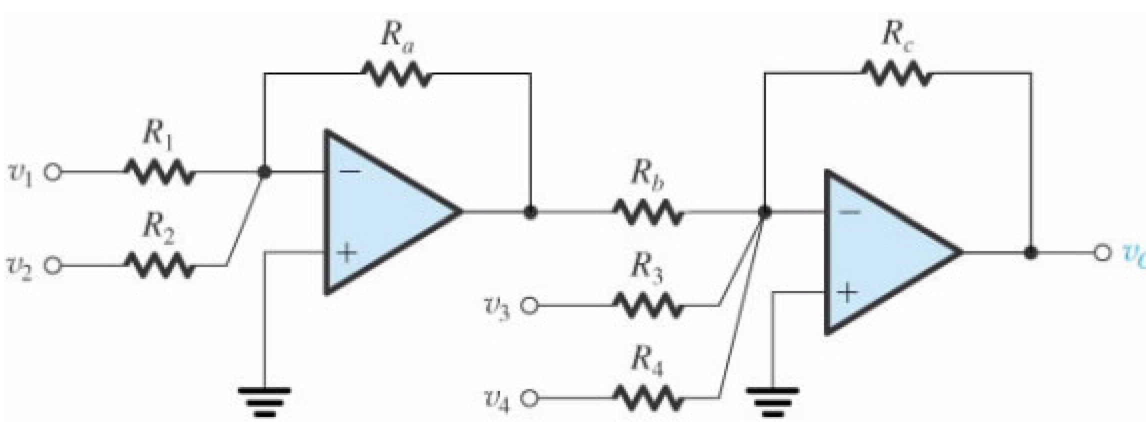


$$i_1 = \frac{v_1}{R_1},\ i_2 = \frac{v_2}{R_2},\ \ldots,\ i_n = \frac{v_n}{R_n}$$

$$i = i_1 + i_2 + \cdots + i_3$$

$$v_O = 0 - iR_f$$

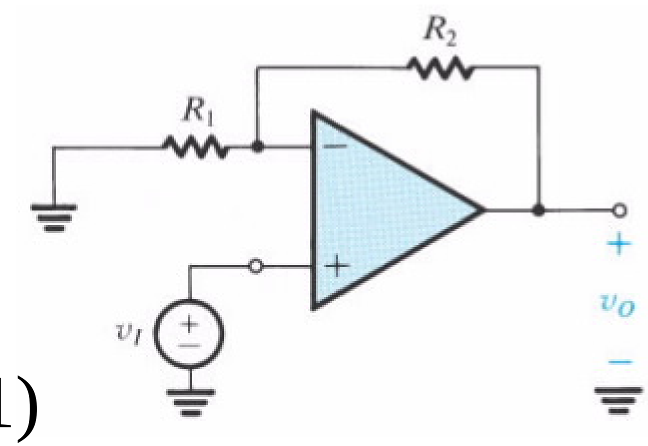
$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \cdots + \frac{R_f}{R_n}v_n\right)$$



RRRRRR dcaccc 1234 RRRRRR 1234 bb **(2.8)**

Non-inverting configuration

- No inversion !
  - Inverting conf: G = - R2/R1



- Virtual short circuit (v2 = v1)

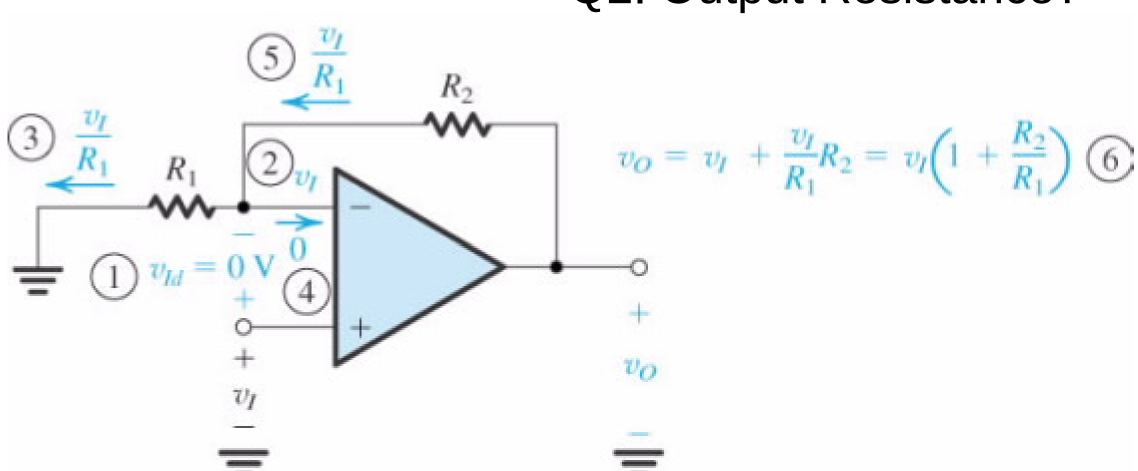
$Id \quad \frac{0}{A} \quad 0$ **for A =**

Q1. Input Resistance?
Q2. Output Resistance?

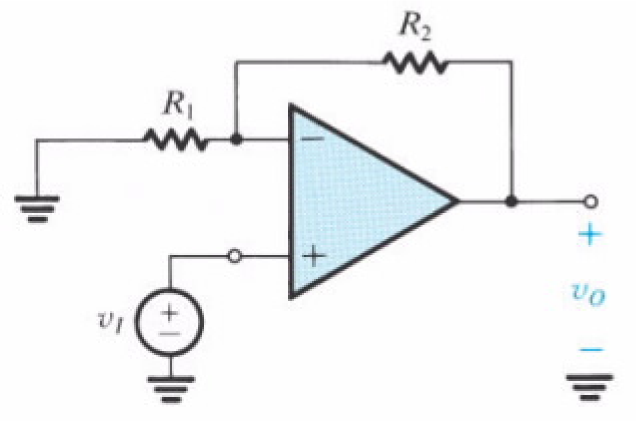
$02 \quad I \quad \frac{I}{R_1} \quad R$

Gain $\frac{0}{I} \quad 1 \quad \frac{R_2}{R_1}$

$10 \quad \frac{R_1}{RR_{12}}$



Finite open loop gain



$$v_o = v_2 + \frac{v_2}{R_1} R_2 = (1 + \frac{R_2}{R_1}) v_2$$

$$v_2 = v_o / (1 + R_2 / R_1)$$

$$v_o = A(v_I - v_2) = A(v_I - \frac{v_o}{(1 + R_2/R_1)})$$

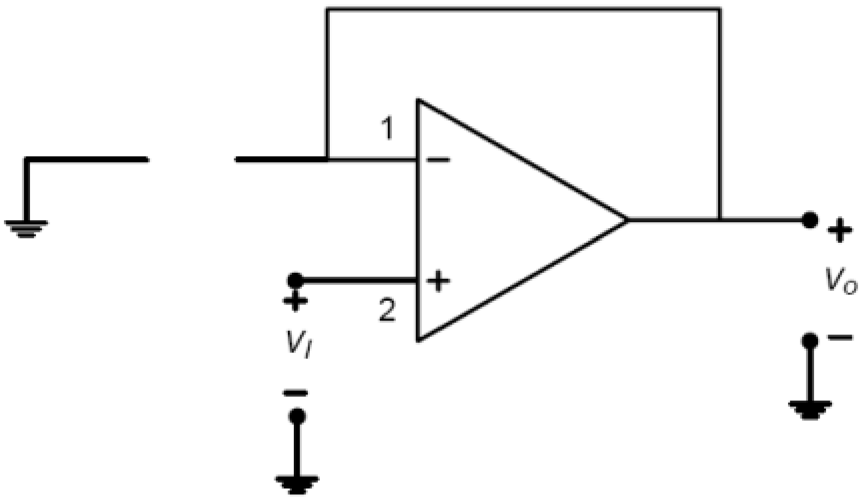
$$A v_I = (1 + \frac{A}{(1 + R_2/R_1)}) v_o$$

$$G \equiv \frac{v_o}{v_I} = A / (1 + \frac{A}{(1 + R_2/R_1)})$$

$$= \boxed{\frac{1 + R_2/R_1}{1 + (1 + R_2/R_1)/A}}$$

- If A >> 1+R2/R1 ➔ G = 1+R2/R1

Voltage follower (unity buffer)



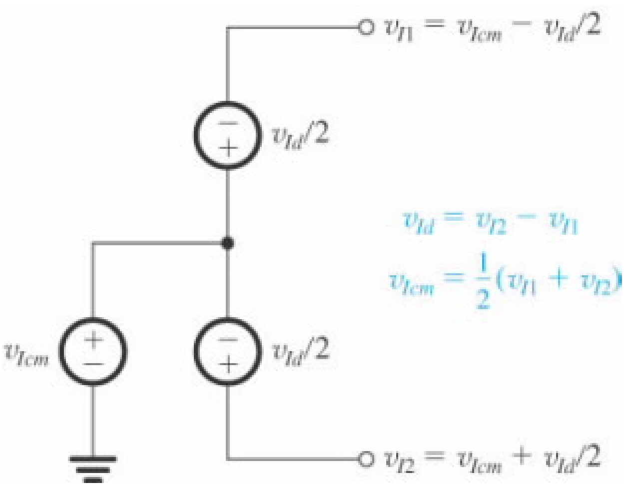
Now what happens as $R_1 \rightarrow$ infinity and $R_f \rightarrow 0$

$$\frac{v_O}{v_I} = 1 + \frac{0}{\infty} \rightarrow 1$$

→ Unity-Gain Amplifier

→ Useful for buffering between stages

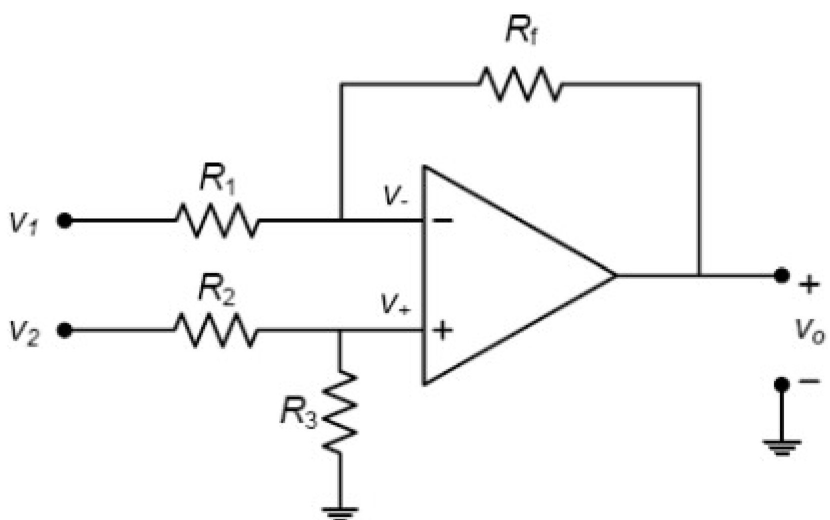
Difference Amplifier (two sources)



$0 \quad AA_{dIdcmIcm}$ **(2.13)**

- Common mode rejection ratio:

$$\mathbf{CMRR} = 20\log\frac{|\mathbf{A}_d|}{|\mathbf{A}_{cm}|} \qquad \mathbf{(2.1 \qquad 4)}$$

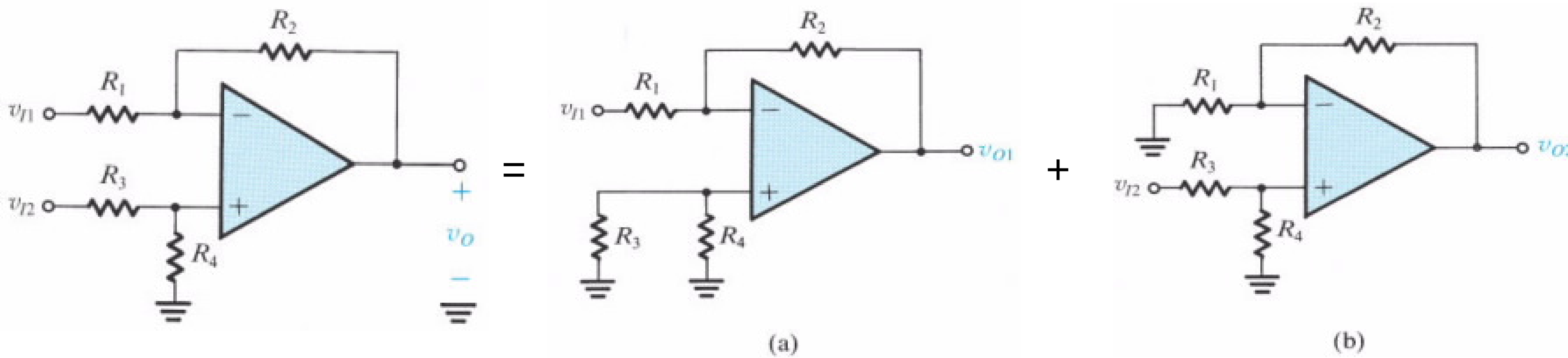


Solution: $\frac{R_3}{R_2} = \frac{R_f}{R_1}$

Analysis either by

- Brute force (힘으로~)
- Superposition (머리로~)

Example (Superposition): Single Diff. Amp.

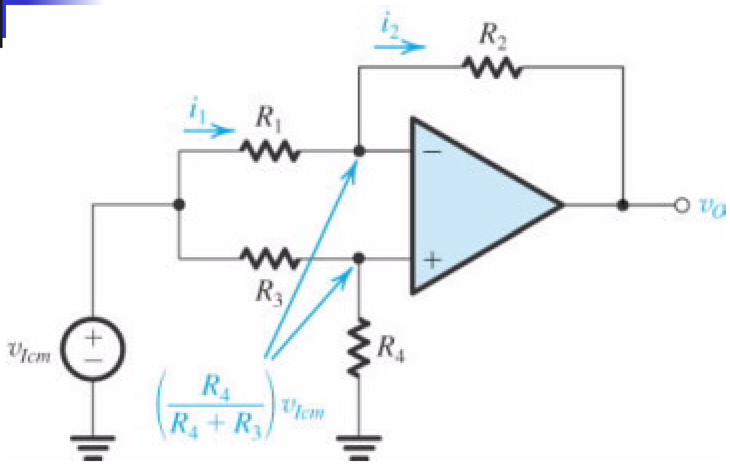


$$v_{O1} = -\frac{R_2}{R_1} v_{I1} \qquad v_{O2} = \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right) v_{I2}$$

By superposition: $v_O = \frac{R_2}{R_1}(v_{I2} - v_{I1}) = A_d v_{Id}$ **(2.16)**

Differential gain: $A_d = \frac{R_2}{R_1}$ **(2.17)**

Usual selection: $R_3 = R_1$ **and** $R_4 = R_2$



$$i_1 \quad \frac{1}{R_1 R_4 R_3} \; Icm Icm \; \frac{R_4}{\quad}$$

$$Icm \; \frac{R_3}{R_4 R_3 R_1} \; \frac{1}{\quad} \quad (2.18)$$

$$OIcm \; \frac{R_4}{R_4 R_3} \qquad i_2 R_2$$

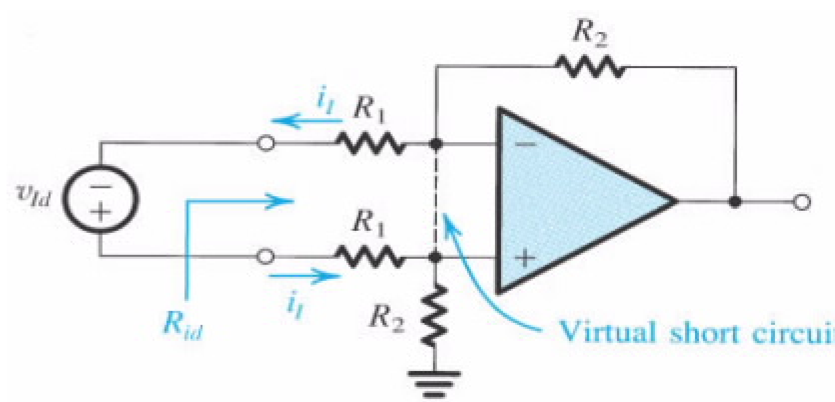
$$OIcmIcm \; \frac{R_2 R_4}{R_4 R_3 R_1 R_4 R_3} \quad \frac{\quad}{\quad} \; \frac{R_3}{\quad}$$

$$= 1 \; \frac{R_2 R_4}{R_4 R_3 R_1 R_4} \quad \frac{\quad}{\quad} \; \frac{R_3}{\quad} \quad Icm$$

$$A_{cm} \; 1 \quad \frac{O}{Icm} \; (2.19) \; \frac{R_2 R_4}{R_4 R_3 R_1 R_4} \quad \frac{\quad}{\quad} \; \frac{R_3}{\quad}$$

A_{cm} 0

Good!



$$R_{id} \; R \quad 2 \quad {}_1 \qquad (2.20)$$

Problem: Low input resistance (see 2.4.2)

Summary

Characteristic of the ideal Op Amp
(Open loop)

1. Infinite input impedance
2. Zero output impedance
3. Zero common-mode gain or, equivalently, infinite common-mode rejection
4. Infinite open-loop gain A
5. Infinite bandwidth
6. Ideal voltage controlled voltage source

* Finite open loop gain (A) should also be noted.
But in most cases, infinite gain model is enough.

Characteristic of the ideal Op Amp
(Closed loop – feedback)

1. Inverting configuration
   G = -R2/R1, Rin = R1, Ro = 0
   Applications: summer, integrator, differentiator, …
- Non-inverting configuration
  G = 1+R2/R1, Rin = inf., Ro = 0
  Applications: unity buffer …
- Difference amp. (R2/R1 = R4/R3)
  G = R2/R1, CMRR=inf.
  Rin = 2*R1 (if R1=R3, R2=R4), Ro = 0